

Proper Pseudoholomorphic Maps between Strictly Pseudoconvex Regions

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1. Introduction

The regularity up to the boundary of a proper holomorphic map between strictly pseudoconvex bounded domains D and D' of \mathbb{C}^n has been widely studied when $n \geq 2$ and is now as well understood as the one-dimensional case. A continuous map $F: D \rightarrow D'$ is said to be *proper* if $F^{-1}(K)$ is compact for every compact set K in D' . If D and D' have C^r -boundaries ($r \geq 2$) then such a map F has a $C^{r-1/2}$ -extension to the boundary, and this is the maximal regularity [4; 20] that can be expected. Various authors have contributed to this result. We just mention Fefferman, who proved in 1974 that if D and D' have smooth boundaries and if F is a biholomorphism, then F extends smoothly to the boundary [9]. We refer to the survey of Forstnerič [10] for a thorough history.

Our aim here is to study the boundary behavior of proper pseudoholomorphic maps between strictly pseudoconvex regions in almost complex manifolds. In order to establish an analogue of the known result in the complex situation, we will need to adapt objects and tools specific to the integrable case. Note, for example, that there is no longer a notion of an analytic set and that the Jacobian of a pseudoholomorphic map is not pseudoholomorphic. Our method uses pseudoholomorphic discs. Originally introduced by E. Bishop, this method has provided geometric proofs of various versions of Fefferman's theorem [17; 25] even in the almost complex case [5; 12].

We consider the following situation. Let D be a bounded domain in some smooth (real) manifold, and let J be an almost complex structure of class C^1 on \bar{D} that is smooth in D . Throughout this paper, we will say that (D, J) is a *strictly pseudoconvex region* if D is defined by $\{\rho < 0\}$, where ρ is a C^2 -regular defining function that is strictly J -plurisubharmonic on \bar{D} . We say (D, J) is a *strictly pseudoconvex region of class C^r* when ρ and J are at least of class C^r . In the complex situation, the regularity is thus the regularity of the boundary.

The first step of our proof is to derive the Hölder $\frac{1}{2}$ -continuous extension, which comes from a sized estimate of the set of regular values and from estimates of the Kobayashi metric. To obtain more regularity, the main obstacle compared with the biholomorphic case is obviously the existence of critical points. Thus we begin with studying the locus of all these points. For the complex case, Pinchuk [19; 20]