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Interpolation by Entire Functions with Growth Conditions

Myriam Ounaïes

0. Introduction

Let $p\colon \mathbb{C} \to [0, +\infty[$ be a weight (see Definition 1.1) and let $A_p(\mathbb{C})$ be the vector space of all entire functions satisfying $\sup_{z\in\mathbb{C}}|f(z)| \le \exp(-Bp(z)) < \infty$ for some constant $B > 0$. For instance, if $p(z) = |z|$, then $A_p(\mathbb{C})$ is the space of all entire functions of exponential type.

Following [3], the interpolation problem we are considering is as follows. Let $V = \{(z_j, m_j)\}_j$ be a multiplicity variety; that is, suppose $\{z_j\}_j$ is a sequence of complex numbers diverging to ∞, $|z_j| \le |z_{j+1}|$, and $\{m_j\}_j$ is a sequence of strictly positive integers. Let $\{w_{j,l}\}_{j,0\le l<m_j}$ be a doubly indexed sequence of complex numbers.

Under what conditions does there exist an entire function $f \in A_p(\mathbb{C})$ such that

$$\frac{f^{(l)}(z_j)}{l!} = w_{j,l} \quad \forall j,\ \forall 0 \le l < m_j?$$

In other words, if we denote by ρ the restriction operator defined on $A_p(\mathbb{C})$ by

$$\rho(f) = \left\{\frac{f^{l}(z_j)}{l!}\right\}_{j,\,0\le l<m_j},$$

what is the image of $A_p(\mathbb{C})$ by ρ?

We say that V is an *interpolating variety* when $\rho(A_p(\mathbb{C}))$ is the space of all doubly indexed sequences $W = \{w_{j,l}\}$ satisfying the growth condition

$$|w_{j,l}| \le A\exp(Bp(z_j)) \quad \forall j,\ \forall 0 \le l < m_j,$$

for certain constants $A, B > 0$.

We have the following important result.

Theorem 0.1 [2, Cor. 4.8]. *V is an interpolating variety for $A_p(\mathbb{C})$ if and only if, for some constants $A, B > 0$, the following conditions hold*:

(i) *for all $R > 0$, $N(0,R) \le AP(R) + B$*;
(ii) *for all $j \in \mathbb{N}$, $N(z_j, |z_j|) \le AP(z_j) + B$.*

Here, $N(z, r)$ denotes the integrated counting function of V in the disc of center z and radius r (see Definition 1.3).

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