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Holomorphic Maps on Projective Spaces and Continuations of Fatou Maps

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1. Introduction

Let f be a holomorphic map of the complex projective space $\mathbb{P}^n$ of dimension n onto itself. We assume that f is of degree $d \geq 2$. As in the one-dimensional case, the Fatou set Ω for f is defined by

$$\Omega := \{p \in \mathbb{P}^n \mid \{f^j\}_{j \geq 0} \text{ is a normal family on a neighborhood of } p\}.$$

It is sometimes useful to consider the normality of the sequence $\{f^j\}_{j\geq 0}$ when restricted to local analytic sets of lower dimension. We introduce a slightly more general concept as follows.

Definition. A holomorphic map φ from a complex analytic space R into $\mathbb{P}^n$ is said to be a *Fatou map* for f if $\{f^j \circ \varphi\}_{j\geq 0}$ is a normal family.

This definiton was given in [U3] and [R] independently. The following facts are immediate. An open set V in $\mathbb{P}^n$ is contained in the Fatou set Ω if and only if the inclusion map $V \hookrightarrow \mathbb{P}^n$ is a Fatou map. If $\varphi\colon R \to \mathbb{P}^n$ is a holomorphic map with $\varphi(R) \subset \Omega$, then φ is a Fatou map.

In this paper we prove the following two theorems.

Theorem 1. *Let S be a Riemann surface and let E be a closed polar set in S. If $\varphi\colon S - E \to \mathbb{P}^n$ is a Fatou map, then φ can be extended to a Fatou map $\check{\varphi}\colon S \to \mathbb{P}^n$.*

Theorem 2. *Let S be a Riemann surface and let E be a closed polar set in S. If $\varphi\colon S \to \mathbb{P}^n$ is a holomorphic map and if $\varphi(S - E)$ is contained in the Fatou set Ω, then $\varphi(S)$ is also contained in Ω.*

Here E is said to be *polar* if it is locally expressed as the set on which a subharmonic function takes the value $-\infty$.

Theorem 1 may be viewed as an analogue of theorems concerning continuation of a holomorphic map from $S - E$ into a compact Riemann surface of genus ≥ 2 [N; Su1] or, more generally, into a complex manifold whose universal cover is a bounded domain of certain type [Su2]. Our proof is an adaptation of Suzuki's idea.

As an application of these theorems, we make a remark concerning a result due to Fornæss and Sibony [FS3]. A connected component of the Fatou set Ω is called

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