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Extremal Discs and Analytic Continuation of Product CR Maps

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Introduction

One of the essentially multidimensional phenomena in complex analysis is the forced analytic continuation of a germ of a biholomorphic map $M_1 \to M_2$ between real analytic manifolds M_1 and M_2 in $\mathbf{C}^n$, $n > 1$. Poincaré (1907) observed that a biholomorphic map sending an open piece of the unit sphere in $\mathbf{C}^2$ to another such open piece must be an automorphism of the unit ball. This was proved for $\mathbf{C}^n$ by Tanaka (1962) and then rediscovered by Alexander [A].

Pinchuk [P] proved that, if M_1 and M_2 are strictly pseudoconvex real analytic nonspherical hypersurfaces and M_2 is compact, then a germ of a biholomorphic map $M_1 \to M_2$ holomorphically extends along any path in M_1. Ezhov, Kruzhilin, and Vitushkin [EKV] gave a different proof of that result. Webster [W] proved that a germ of a biholomorphic map $M_1 \to M_2$ between real algebraic Levi nondegenerate hypersurfaces in $\mathbf{C}^n$ is algebraic.

There is an impressive number of publications in which M_1 and M_2 are real *algebraic* manifolds of different dimensions or higher codimension, in particular real quadratic manifolds (see [BER]). Hill and Shafikov [HS] proved the analytic continuation result in higher codimension where only one of the manifolds M_1 and M_2 is assumed to be algebraic. There are many more results on the problem that we omit here (see e.g. [BER; HS] for references).

Despite the large amount of work done on the problem, there seem to be no results in the literature where M_1 and M_2 are manifolds of higher codimension in $\mathbf{C}^n$ and neither of them is algebraic. In this paper we consider the case in which M_1 is a real analytic strictly pseudoconvex manifold and M_2 is the Cartesian product of several compact strictly convex real analytic hypersurfaces. In particular, we give another proof of Pinchuk's [P] result for the case where M_2 is strictly convex and neither of the M_j is assumed to be nonspherical.

For the case in which M_2 is the product of two spheres, the result was obtained earlier by the first author [Sc]. In this paper we significantly simplify and generalize the proof given in [Sc]. Following [Sc], we use a new method based on extremal discs in higher codimension. As by-products, we obtain some properties of extremal discs that may be useful elsewhere.

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