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On the Solid Hull of the Hardy Space H^p, $0 < p < 1$

Miroljub Jevtić & Miroslav Pavlović

1. Introduction

Finding the solid hull $S(H^p)$ of the Hardy space H^p—that is, finding the strongest growth condition the absolute value of the coefficients of H^p functions must satisfy—is an old and difficult problem. It follows from Littlewood's theorem on random power series [7, Thm. A.5, p. 228] that $S(H^p) = H^2$ for $2 < p < \infty$. Much later, Kisliakov [12] identified the solid hull of the space H^∞. A deep result of Kisliakov shows that $S(H^\infty)$ is also H^2. In this paper we identify $S(H^p)$ in the case $0 < p < 1$.

The Hardy space H^p $(0 < p \le \infty)$ is the space of all functions f holomorphic in the unit disc U $(f \in H(U))$ for which

$$\|f\|_p = \lim_{r\to 1} M_p(r, f) < \infty,$$

where, as usual,

$$M_p(r, f) = \left(\frac{1}{2\pi}\int_0^{2\pi} |f(re^{it})|^p \, dt\right)^{1/p}, \quad 0 < p < \infty,$$

and

$$M_\infty(r, f) = \sup_{0\le t<2\pi} |f(re^{it})|.$$

Throughout this paper, we identify a holomorphic function $f(z) = \sum_{n=0}^{\infty} \hat{f}(n) z^n$ with its sequence of Taylor coefficients $(\hat{f}(n))_{n=0}^{\infty}$. Hardy and Littlewood proved that if f belongs to H^p, $0 < p < 1$, then

$$\sum_{n=0}^{\infty} (n+1)^{p-2} |\hat{f}(n)|^p < \infty \tag{1.1}$$

and

$$|\hat{f}(n)| = o((n+1)^{1/p-1}), \quad n \to \infty \tag{1.2}$$

(see [7] for information and references).

In [13] it was proved that if $f \in H^p$, $0 < p < 1$, then

$$\sum_{n=1}^{\infty} 2^{-n(1-p)} \Big(\sup_{0\le k\le 2^n} |\hat{f}(k)|\Big)^p < \infty, \tag{1.3}$$

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