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# Complexes of Nonseparating Curves and Mapping Class Groups

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## 1. Introduction

Let $R$ be a compact, connected, orientable surface of genus $g$ with $p$ boundary components. The mapping class group $\mathrm{Mod}_R$ of $R$ is the group of isotopy classes of orientation-preserving homeomorphisms of $R$. The extended mapping class group $\mathrm{Mod}_R^*$ of $R$ is the group of isotopy classes of all (including orientation-reversing) homeomorphisms of $R$. Let $\mathcal{A}$ denote the set of isotopy classes of nontrivial simple closed curves on $R$. The *complex of curves* $\mathcal{C}(R)$ on $R$ is an abstract simplicial complex, introduced by Harvey [H], with vertex set $\mathcal{A}$ such that a set of $n$ vertices $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$ forms an $(n-1)$-simplex if and only if $\alpha_1, \alpha_2, \dots, \alpha_n$ have pairwise disjoint representatives.

Definition. A simplicial map $\lambda \colon \mathcal{C}(R) \to \mathcal{C}(R)$ is called *superinjective* if the following condition holds: If $\alpha$ and $\beta$ are two vertices in $\mathcal{C}(R)$ such that the geometric intersection number $i(\alpha, \beta)$ of $\alpha$ and $\beta$ is not equal to zero, then $i(\lambda(\alpha), \lambda(\beta))$ is not equal to zero.

The combinatorial structure of curve complexes on surfaces are studied in order to derive information about the algebraic structure of the mapping class groups. In [Iv1], Ivanov proved that if $g \geq 2$ then every automorphism of $\mathcal{C}(R)$ is induced by a homeomorphism of $R$. He proved that $\mathrm{Aut}(\mathcal{C}(R)) \cong \mathrm{Mod}_R^*$ for most surfaces, and as an application he gave a complete description of isomorphisms between finite index subgroups of $\mathrm{Mod}_R^*$. Ivanov proved that every such isomorphism is induced by a homeomorphism of $R$; that is, it is of the form $k \to hkh^{-1}$ for some $h \in \mathrm{Mod}_R^*$ for most surfaces. These theorems were extended to most of the surfaces of genus 0 and 1 by Korkmaz in [K] and independently by Luo in [L2]. Luo gave a proof by using a multiplicative structure on the set of isotopy classes of nonseparating simple closed curves on $R$, a structure introduced by him in [L1].

Ivanov and McCarthy [IvM] gave a complete description of injective homomorphisms between mapping class groups of surfaces $\mathrm{Mod}_R$ and $\mathrm{Mod}_{R'}$ when the maxima of ranks of abelian subgroups of $\mathrm{Mod}_R$ and $\mathrm{Mod}_{R'}$ differ by at most 1. In particular they showed that, for most surfaces, an injective homomorphism of $\mathrm{Mod}_R$ to itself is of the form $k \to hkh^{-1}$ for some $h \in \mathrm{Mod}_R^*$.

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