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Extremal Bases and Hölder Estimates for $\bar\partial$ on Convex Domains of Finite Type

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1. Introduction

Let $D \subset\subset \mathbb{C}^n$ be a smoothly bounded convex domain of finite type in the sense of D'Angelo [D'A]. The main object of this paper is to compare two different pseudo-distances and related geometric constructions describing the complex geometry of the boundary bD of D. These constructions originated with the work of McNeal on complex geometry and on the Bergman kernels of convex domains of finite type [Mc1; Mc2]; today they have taken an important place also in the study of other function-theoretic questions on convex domains of finite type, especially for problems in which the geometric shape of the domain is to be precisely measured and translated into quantitative function-theoretic information. Prominent examples of such problems are the characterization of the zero sets of functions in the Nevanlinna class as well as finding optimal estimates for the inhomogeneous Cauchy–Riemann equation $\bar\partial u = f$ in D, where f is a $\bar\partial$-closed differential form of bidegree $(0, q+1)$ belonging to some function space—for example, to $L^p_{0,q+1}(D)$, $1 \le p \le \infty$. Our results should be seen in this context, although some of them can also be viewed as purely geometric statements about smooth convex domains of finite type that may be interesting in their own right.

The main application that we will be concerned with in this paper is the problem of (optimal) Hölder estimates for the $\bar\partial$-equation with essentially bounded right-hand side. We do not attempt to give complete references for the other problems already mentioned, instead referring the interested reader to the work of McNeal [Mc1; Mc2], McNeal and Stein [McS], Bruna, Charpentier, and Dupain [BCDu], Diederich and Mazzilli [DM], and Cumenge [Cu3]. A different approach to Hölder estimates for the $\bar\partial$-operator from the one pursued here is described in the work of Cumenge [Cu1; Cu2].

The starting point to motivate our investigations is the following theorem of Diederich, Fischer, and Fornæss [DFiFo] and Cumenge [Cu1].

1.1. Theorem. *Let $D \subset\subset \mathbb{C}^n$ be a convex domain with $\mathcal{C}^\infty$-smooth boundary of finite type m. Then there exist bounded linear integral operators*

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