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A Few Weight Systems Arising from Intersection Graphs

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1. Introduction

Finite-type invariants have received much attention over the past decade. One reason for this is that they provide a common framework for many of the most powerful knot invariants, such as the Conway, Jones, HOMFLYPT, and Kauffman invariants. The framework also allows us to study these invariants using elementary combinatorics, by looking at associated functionals (called *weight systems*) on spaces of *chord diagrams.* This provides new ways of describing the invariants.

The modest goal of this paper is to define a few weight systems in terms of the adjacency matrix of the intersection graph of the chord diagrams, and to show that among these weight systems are those associated with the Conway, HOMFLYPT, and Kauffman polynomials in both their framed and unframed incarnations. This gives us new formulas for the weight systems associated to these important knot invariants. We build on ideas of Bar-Natan and Garoufalides [2], who first found the formula we give for the Conway polynomial.

In Section 2 we will review the necessary background for the paper: finite-type invariants, the 2-term relations introduced by Bar-Natan and Garoufalides, intersection graphs of chord diagrams, and Lando's graph bialgebra. In Section 3 we will study the adjacency matrix of the intersection graph; we show that the weight systems associated with the Conway and HOMFLYPT polynomials can be defined in terms of the determinant and rank of this matrix. In Section 4 we look at *marked* chord diagrams and define an extended set of 2-term relations on these diagrams. We give an explicit set of generators for the space of marked chord diagrams modulo these relations. Finally, we show that the weight system associated with the Kauffman polynomial can be defined in terms of the rank of the adjacency matrix of marked chord diagrams.

Remark. The result for the Conway polynomial (Theorem 4) has already been proved by Bar-Natan and Garoufalidis [2] but is included here for completeness and to place it in the context of Lando's bialgebra. After distributing the first version of this paper [11], the author discovered that the adjacency matrix of an intersection graph has also been studied by Soboleva [15], who has also proven Theorem 5 and a weaker version of Theorem 11. The intersection graphs we study are also related to the *trip matrix* of a knot, studied by Zulli [18].

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