Michigan Math. J. 51 (2003)

Bounds on the Average Bending of the Convex Hull Boundary of a Kleinian Group

Martin Bridgeman

1. Introduction

In this paper we consider hyperbolic manifolds with incompressible convex core boundary. We show that total bending along a geodesic arc on the boundary of the convex core is bounded above by a function of its length. Integrating this function over the unit tangent bundle of the boundary of the convex core, we obtain a new universal upper bound on the total bending of the convex core boundary. Furthermore, we produce a new universal upper bound on the Lipschitz constant for the map from the convex core boundary to the hyperbolic structure at infinity. These results improve on earlier bounds of Bridgeman and Canary.

Let $N = \mathbf{H}^3/\Gamma$ be an orientable hyperbolic manifold with domain of discontinuity $\Omega(\Gamma)$ and limit set L_Γ. In this paper we restrict ourselves to the case when all the components of $\Omega(\Gamma)$ are simply connected. This is a natural restriction to make and includes the set of quasi-Fuchsian groups. Let $CH(L_\Gamma)$ be the convex hull of Γ and let β_Γ be the bending lamination on $\partial CH(L_\Gamma)$. Let $C(N) = CH(L_\Gamma)/\Gamma$ be the convex core and let β_N be the bending lamination on $\partial C(N)$. Then we observe that $\partial C(N)$ is incompressible if and only if the components of $\Omega(\Gamma)$ are all simply connected.

If α is a geodesic arc in $CH(L_\Gamma)$ then the average bending $B(\alpha)$ is defined to be the bending per unit length, or specifically

$$B(\alpha) = \frac{i(\alpha, \beta_\Gamma)}{l(\alpha)},$$

where i is the intersection number and $l(\alpha)$ is the length of α (see [2]).

In [2], Bridgeman considers bounds on the average bending for quasi-Fuchsian groups and proves that, for a quasi-Fuchsian group Γ, if $l(\alpha) \leq \log 3$ then $i(\alpha, \beta_\Gamma) \leq 2\pi$. In [3], the geometry of the convex core boundary $\partial C(N)$ is compared with the geometry of the domain of discontinuity Ω/Γ for a general Kleinian group. One outcome is an improvement of the bound just described on intersection number to prove that, for a Kleinian group Γ such that the components of $\Omega(\Gamma)$ are simply connected, if $l(\alpha) \leq 2\sinh^{-1} 1$ then $i(\alpha, \beta_\Gamma) \leq 2\pi$.

Both these bounds on the intersection number give universal upper bounds for the average bending of geodesic arcs of a given fixed length. By considering geodesics α of length $l(\alpha) = 2\sinh^{-1} 1$, we obtain $B(\alpha) \leq \pi/\sinh^{-1} 1$.

Received February 25, 2002. Revision received August 23, 2002.