Michigan Math. J. 47 (2000)

The Interaction of Dirac Particles with Non-Abelian Gauge Fields and Gravity—Black Holes

Felix Finster, Joel Smoller, & Shing-Tung Yau

1. Introduction

Recently, the Einstein–Dirac–Yang/Mills (EDYM) equations were derived for a static, spherically symmetric system of a Dirac particle that interacts with both a gravitational field and the magnetic component of an SU(2) Yang/Mills field [5]. This system was constructed by choosing a representation of the rotation group that acts nontrivially on the YM index and by making an *ansatz* involving two real functions for the Dirac wave function, which is invariant under this group representation. The resulting equations were shown to admit stable particle-like solutions for physically relevant values of the coupling constants. In this paper, we study black-hole solutions of these EDYM equations.

We prove that the only black-hole solutions of our EDYM equations are the Bartnik–McKinnon (BM) black holes; that is, the spinors must vanish identically. In other words, the EDYM equations do not admit normalizable black-hole solutions. Thus, in the presence of quantum mechanical Dirac particles, static and spherically symmetric black-hole solutions do not exist. Another interpretation of our result is that Dirac particles can only either disappear into the black hole or escape to infinity. These results are proved under very weak regularity assumptions on the form of the event horizon; see assumptions (I)–(III) in the next section.

Our work here is a continuation of [3], where we showed that the Einstein–Dirac–Maxwell (EDM) equations do not admit normalizable black-hole solutions. However, in contrast to the EDM system [3], where an electric field is present, here we consider the influence of a magnetic field—more precisely, the influence of the magnetic component of a non-abelian gauge field. Furthermore, we point out that the Dirac particle considered here has zero total angular momentum. In analogy to [3], one could also form a spherically symmetric system out of $(2j+1)$ Dirac particles each having angular momentum j, $j = 1, 2, \dots$. However, the EDYM equations for such a system would involve four real spinor functions. This is a considerably more difficult problem, which we are presently investigating.

Received October 13, 1999. Revision received December 6, 1999.
Research of J. Smoller supported in part by NSF Grant no. DMS-G-9802370. Research of S.-T. Yau supported in part by NSF Grant no. 33-585-7510-2-30.