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Logarithmic Series and Hodge Integrals in the Tautological Ring

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(with an Appendix by D. Zagier)

Dedicated to William Fulton on the occasion of his 60th birthday

0. Introduction

0.1. Overview. Let $\bar{M}_g$ be the moduli space of Deligne–Mumford stable curves of genus $g \geq 2$. The study of the Chow ring of the moduli space of curves was initiated by Mumford in [Mu]. In the past two decades, many remarkable properties of these intersection rings have been discovered. Our first goal in this paper is to describe a new perspective on the intersection theory of the moduli space of curves that encompasses advances from both classical degeneracy studies and topological gravity. This approach is developed in Sections 0.2–0.7.

The main new results of the paper are computations of basic Hodge integral series in $A^*(M_g)$ encoding the canonical evaluations of $\kappa_{g-2-i}\lambda_i$. The motivation for the study of these tautological elements and the series results are given in Section 0.8. The body of the paper contains the Hodge integral derivations.

0.2. Moduli Filtration. We will consider the moduli filtration

$$\bar{M}_g \supset M_g^c \supset M_g \supset \{[X_g]\}. \tag{1}$$

Here, X_g is a fixed nonsingular curve, M_g is the moduli space of nonsingular genus g curves, and M_g^c is the moduli space of stable curves of compact type (curves with tree dual graphs or, equivalently, with compact Jacobians).

Let $A^*(\bar{M}_g)$ denote the Chow ring with $\mathbb{Q}$-coefficients. Intersection theory on $\bar{M}_g$ may be naturally viewed in four stages corresponding to the filtration (1). There is an associated sequence of successive quotients:

$$A^*(\bar{M}_g) \to A^*(M_g^c) \to A^*(M_g) \to A^*([X_g]) \cong \mathbb{Q}. \tag{2}$$

We develop here a uniform approach to the study of these quotient rings.

0.3. Tautological Rings. The study of the structure of the entire Chow ring of the moduli space of curves appears quite difficult at present. While presentations are known in a few genera [F1; F2; I; Mu], no general results have yet been conjectured. Since the principal motive is to understand cycle classes obtained from algebro-geometric constructions, it is natural to restrict inquiry to the *tautological ring* $R^*(\bar{M}_g) \subset A^*(\bar{M}_g)$.

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