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Maximal Surfaces of Riemann Type in Lorentz–Minkowski Space $\mathbb{L}^3$

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1. Introduction and Statement of Results

In 1867, Riemann [19] found a 1-parameter family of complete minimal surfaces in the 3-dimensional Euclidean space $\mathbb{E}^3$ that are fibered by circles and straight lines in parallel planes. Riemann also proved that these are the only surfaces (besides the catenoid) with this property. In 1870, Enneper [3] proved that if a minimal surface in $\mathbb{E}^3$ is foliated by pieces of circles, then the planes containing these circles are actually parallel and so the surface is a piece of either a Riemann example or a catenoid. Nowadays, we know more general uniqueness theorems for Riemann minimal examples (see e.g. [4; 10; 14]).

In this paper we deal with the same kind of questions for maximal spacelike surfaces in Lorentz–Minkowski 3-dimensional space $\mathbb{L}^3$. A smooth immersion of a surface in $\mathbb{L}^3$ is called *spacelike* if the induced metric on the surface is a Riemannian metric. A spacelike surface in $\mathbb{L}^3$ is maximal provided its mean curvature vanishes. Spacelike maximal surfaces in $\mathbb{L}^3$ represent a maximum for the area integral [1]. It is known that the only complete maximal spacelike surfaces are planes (see [1] and [2] for arbitrary dimension). Hence, it is natural to consider nonflat maximal spacelike immersions with singularities. These singularities correspond to either curves of points where the immersion is not spacelike or to isolated branch points.

Some properties of minimal surfaces in $\mathbb{E}^3$ have an analogous version for maximal spacelike surfaces in $\mathbb{L}^3$. For example, they admit a Weierstrass representation closely related to that of minimal surfaces in $\mathbb{E}^3$. As a matter of fact, there is a natural method of constructing maximal surfaces in $\mathbb{L}^3$ from minimal ones in $\mathbb{E}^3$, and vice versa.

Inspired by the works of Riemann and Enneper just cited, we classify maximal spacelike surfaces in $\mathbb{L}^3$ that are foliated by pieces of circles. Rotational maximal surfaces in $\mathbb{L}^3$ have been studied in [7]. As in the minimal case in $\mathbb{E}^3$, a maximal spacelike surface in $\mathbb{L}^3$ is foliated by circles in parallel planes if and only if a Shiffman-type function vanishes at any point of the surface. This function lies in the kernel of the Lorentzian Jacobi operator of the surface. In this work we prove

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