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Stable Compactifications of Polyhedra

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1. Introduction

To set the stage, we begin with some definitions.

Definition 1.1. (i) If X is a compact metric space and $Z \subset X$ is closed, then Z is said to be a *Z-set* if there is a homotopy $h_t : X \to X$ $(0 \le t \le 1)$ such that $h_0(x) = x$ for all x and $h_t(X) \subset X - Z$ for all $t > 0$. The model case is that in which X is a topological manifold and $Z = \partial X$. Another interesting case is the visual compactification of a CAT(0) space.

(ii) A separable metric space X is said to be an *ANR* if X can be embedded in separable Hilbert space in such a way that there is an open neighborhood U of X that retracts to X. All locally contractible finite-dimensional metric spaces are ANRs.

(iii) The *Hilbert cube* I^∞ is defined to be the product $\prod_{i=1}^{\infty}[0,1]$. A *Hilbert cube manifold* X is a separable metric space such that each point in X has an open neighborhood that is homeomorphic to an open subset of the Hilbert cube. Fundamental work of Chapman and West shows that every Hilbert cube manifold is the product of a locally finite polyhedron with I^∞ and that, for a given Hilbert cube manifold, the polyhedron is unique up to simple homotopy.

(iv) If X is a locally compact ANR, then a compact metric space $\bar{X}$ containing X is said to be a *$\mathcal{Z}$-compactification* of X if $Z = \bar{X} - X$ is a $\mathcal{Z}$-set in $\bar{X}$. It follows easily from the definition of $\mathcal{Z}$-set and Hanner's criterion for ANR-ness [10] that, in this case, $\bar{X}$ is also an ANR.

(v) If $\{(K_i, \alpha_i)\}_{i=1}^{\infty}$ is a sequence of finite CW complexes K_i and maps $\alpha_i : K_i \to K_{i-1}$, then the *inverse mapping telescope* $\mathrm{Tel}(K_i, \alpha_i)$ is obtained from the disjoint union of the mapping cylinders of the α_i by identifying the top of the mapping cylinder of α_i with the base of the mapping cylinder of α_{i+1}.

In [4], Chapman and Siebenmann gave necessary and sufficient conditions for a noncompact Hilbert cube manifold X to admit a $\mathcal{Z}$-compactification. Stated geometrically, their condition was that X admits a $\mathcal{Z}$-compactification if and only if X is homeomorphic to the product of an inverse mapping telescope with the Hilbert cube. In the same paper it was asked whether a locally finite polyhedron X admits a $\mathcal{Z}$-compactification whenever $X \times Q$ admits a $\mathcal{Z}$-compactification.

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