Weak Singularity Spectra of the Patterson Measure for Geometrically Finite Kleinian Groups with Parabolic Elements

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1. Introduction and Statement of Results

In this paper we give a multifractal description of the Patterson measure μ supported on the limit set $L(G)$ of a geometrically finite Kleinian group G with parabolic elements. More precisely, we estimate the *weak singularity spectra* of μ, which means that for $\theta > 0$ we determine the Hausdorff dimensions of the following sets:

$$\mathcal{I}^{\theta}(\mu) := \left\{ \xi \in L(G) : \liminf_{r \to 0} \frac{\log \mu(B(\xi, r))}{\log r} \leq \theta \right\},$$

$$\mathcal{I}_{\theta}(\mu) := \left\{ \xi \in L(G) : \liminf_{r \to 0} \frac{\log \mu(B(\xi, r))}{\log r} \geq \theta \right\},$$

$$\mathcal{S}^{\theta}(\mu) := \left\{ \xi \in L(G) : \limsup_{r \to 0} \frac{\log \mu(B(\xi, r))}{\log r} \leq \theta \right\},$$

$$\mathcal{S}_{\theta}(\mu) := \left\{ \xi \in L(G) : \limsup_{r \to 0} \frac{\log \mu(B(\xi, r))}{\log r} \geq \theta \right\},$$

where $B(\xi, r)$ denotes the Euclidean ball of radius r centered at ξ.

This "weak multifractal analysis" of the Patterson measure will be based on a further investigation of the Hausdorff dimension $\dim_H(\mathcal{J}_\sigma(G))$ of the associated *σ-Jarník limit sets* $\mathcal{J}_\sigma(G) \subset L(G)$, which represent the natural generalization of the well-approximable real numbers to the theory of Kleinian groups ($\mathcal{J}_\sigma(G)$ is defined at the end of this section).

In [12] we derived a complete description of $\mathcal{J}_\sigma(G)$ in terms of the dimension with respect to μ. As a consequence, we were able to determine $\dim_H(\mathcal{J}_\sigma(G))$ for those cases in which $\dim_H(L(G))$ does not exceed the maximal rank of the parabolic fixed points of G. The first aim of this paper will be to show how to modify the construction in [12] in order to deal with the remaining cases. That is, based on the construction in [12], we compute $\dim_H(\mathcal{J}_\sigma(G))$ for *all* geometrically finite Kleinian groups with parabolic elements. We then discuss how these estimates

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