On Boundary Regularity of Analytic Discs

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1. Introduction

In this paper we study the boundary behavior of analytic discs near the zero set of a nonnegative plurisubharmonic function or a totally real submanifold of $\mathbb{C}^n$.

Our main result is the following.

THEOREM 1.1. *Let* Ω *be a complex manifold,* ρ *a plurisubharmonic function in* Ω, *and* $f: \Delta \to \Omega$ *a holomorphic map of the unit disc* $\Delta \subset \mathbb{C}$ *into* Ω *such that* $\rho \circ f \geq 0$ *and* $\rho \circ f(\zeta) \to 0$ *as* $\zeta \in \Delta$ *tends to an open arc* $\gamma \subset \partial\Delta$. *Assume that, for a certain point* $a \in \gamma$, *the cluster set* $C(f, a)$ *contains a point* $p \in \Omega$ *such that* ρ *is strictly plurisubharmonic in a neighborhood of* p. *Then* f *extends to a Hölder* $1/2$*-continuous mapping in a neighborhood of* a *on* $\Delta \cup \gamma$. *If, moreover,* $\rho \geq 0$ *and the function* ρ^θ *is plurisubharmonic in a neighborhood of* p *for some* $\theta \in [1/2, 1]$, *then* f *is Hölder* $1/2\theta$*-continuous* (*Lipschitz, if* $\theta = 1/2$) *in a neighborhood of* a *on* $\Delta \cup \gamma$.

Although this result is new even in the case when the function ρ is of class C^∞, we note that ρ is supposed only to be upper semicontinuous. In what follows we write p.s.h. for plurisubharmonic. A function ρ is called *strictly* p.s.h. in a neighborhood of p with local coordinates z if, for some $\varepsilon > 0$, the function $\rho - \varepsilon|z|^2$ is p.s.h. in a neighborhood of p; ρ is called strictly p.s.h. in Ω if it is strictly p.s.h. at each point of Ω.

It seems that the assertion of Theorem 1.1 is new even in the case when Ω is a domain in the complex plane $\mathbb{C}$ (i.e., f is a usual holomorphic function in Δ). Some comments on the conditions of the theorem may be listed as follows.

(1) The manifold Ω cannot be arbitrary because of the existence condition of the described function ρ. For instance, it implies that all the manifolds $\Omega \cap \{\rho < c\}$, $c > 0$, are hyperbolic at the point p by a theorem of Sibony [14].

(2) It is enough to assume that ρ is p.s.h. in a neighborhood of its zero set only. Then, replacing Ω by this neighborhood and f by $f \circ \phi$ where $\phi: \Delta \to V \cap \Delta$ is a conformal mapping for a suitable neighborhood $V \supset \gamma$, we are in the setting of the theorem.

(3) If f is known to be continuous at the point a, then the situation becomes purely local and we can work with Ω as a domain in $\mathbb{C}^n$. But one of the essential

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