Pluripolar Hulls

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1. Introduction

Let E be a pluripolar set in $\mathbb{C}^N$. That is, for each $z_0 \in E$, there exists a neighborhood U of z_0 and a plurisubharmonic (psh) function $u \not\equiv -\infty$ on U with

$$E \cap U \subset \{\, z \in U : u(z) = -\infty \,\}.$$

From the well-known result of Josefson (cf. [K, Thm. 4.7.4]), there exists a plurisubharmonic function u on $\mathbb{C}^N$, $u \not\equiv -\infty$, with $E \subset \{z \in D : u(z) = -\infty\}$. For example, if f is holomorphic in an open set D, then

$$E := \{\, z \in D : f(z) = 0 \,\} = \{\, z \in D : u(z) := \log|f(z)| = -\infty \,\}$$

is pluripolar. It can happen that any psh function u that is $-\infty$ on a pluripolar set $E \subset D$ is automatically $-\infty$ on a larger set. As a simple example, if

$$E = \{\, z \in \mathbb{C}^N : |z_1| < 1,\ z_2 = \cdots = z_N = 0 \,\},$$

then any globally defined psh function u that is $-\infty$ on E must be $-\infty$ on

$$E^* = \{\, z \in \mathbb{C}^N : z_1 \in \mathbb{C},\ z_2 = \cdots = z_N = 0 \,\}.$$

This follows since $U(z_1) := u(z_1, 0, \dots, 0)$ is subharmonic on $\mathbb{C}$ and $-\infty$ on the *nonpolar* set $\{\, z_1 \in \mathbb{C} : |z_1| < 1\}$. To describe this phenomenon of "propagation" of pluripolar sets more concretely, given a pluripolar set E in $\mathbb{C}^N$ and a neighborhood D of E, we define two types of *pluripolar hulls* of E relative to D:

$$E_D^* := \bigcap \{\, z \in D : u(z) = -\infty \,\},$$

where the intersection is taken over *all* psh functions in D that are $-\infty$ on E; and

$$E_D^- := \bigcap \{\, z \in D : u(z) = -\infty \,\},$$

where the intersection is taken over all *negative* psh functions in D that are $-\infty$ on E. Clearly, $E_D^* \subset E_D^-$ and if $E \subset D_1 \subset\subset D_2$ then

$$E_{D_1}^- \subset E_{D_2}^* \cap D_1.$$

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