On a Minimal Lagrangian Submanifold of C^n Foliated by Spheres

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1. Introduction

In general, not much is known about minimal submanifolds of Euclidean space of high codimension. In [1], Anderson studies complete minimal submanifolds of Euclidean space with finite total scalar curvature, trying to generalize classical results of minimal surfaces. More recently, Moore [10] continues the study of this kind of minimal submanifolds.

Harvey and Lawson [6] also study a particular family of minimal submanifolds of complex Euclidean space, the *special Lagrangian* submanifolds—that is, oriented minimal Lagrangian submanifolds. They have the property of being absolutely volume minimizing. Among other things, they construct important examples of the previously mentioned minimal Lagrangian submanifolds. Following their ideas, new examples of this kind of submanifolds are also obtained in [2]. This family is well known in the case of surfaces, because an orientable minimal surface of $\mathbb{C}^2$ is Lagrangian if and only if it is holomorphic with respect to some orthogonal complex structure on $\mathbb{R}^4$ (see [3]).

Among the examples constructed by Harvey and Lawson in [6], we emphasize the one given in Theorem 3.5. In this example, we emphasize one of its connected components, which is defined by

$$\begin{aligned} M_0 = \{(x, y) \in \mathbb{C}^n \equiv \mathbb{R}^n \times \mathbb{R}^n;\ &|x|y = |y|x; \\ &\operatorname{Im}(|x| + i|y|)^n = 1;\ |y| < |x| \tan(\pi/n)\}. \end{aligned}$$

Besides being a minimal Lagrangian submanifold of complex Euclidean space $\mathbb{C}^n$, M_0 is invariant under the diagonal action of $\mathrm{SO}(n)$ on $\mathbb{C}^n \equiv \mathbb{R}^n \times \mathbb{R}^n$. This paper is inspired by this example. We start by showing that it is a very regular example with many similar properties to the classical catenoid. So, from now on we will refer to M_0 as the *Lagrangian catenoid*. Topologically it is $\mathbb{R} \times \mathbb{S}^{n-1}$. Geometrically it is foliated by $(n-1)$-dimensional round spheres of $\mathbb{C}^n$, and it has finite total scalar curvature (see Proposition 1). When $n = 2$ it has total curvature -4π, being one of the examples described by Hoffman and Osserman in [7].

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