Invariant Cauchy–Riemann Operators and Relative Discrete Series of Line Bundles over the Unit Ball of $\mathbb{C}^d$

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0. Introduction

Let G/K be a Hermitian symmetric space realized as a bounded symmetric domain. Shimeno [Sho] gives the Plancherel decomposition for the L^2-space of sections of a homogeneous line bundle over G/K. It is proved that the discrete parts (also called relative discrete series) in the decomposition are all equivalent to holomorphic discrete series. The proof involves identifying the infinitesimal characters of the relative discrete series and those of the holomorphic discrete series. For the unit disk in $\mathbb{C}$, this was proved in [PPZ]. More explicitly, we have found the intertwining operators from the relative discrete series onto the standard modules of the holomorphic discrete series, that is, the Bergman spaces of holomorphic functions; they turn out to be the iterates of the invariant Cauchy–Riemann operator. When G/K is the unit ball in $\mathbb{C}^d$, we have proved [Z] by explicit calculation of the reproducing kernels that the relative discrete series are equivalent to certain weighted Bergman spaces of vector-valued holomorphic functions.

In the present paper we shall give a unified approach to the foregoing results. Here is a brief introduction to the main idea and a summary of the results obtained. Let $\bar{D}$ be the invariant Cauchy–Riemann operator acting on sections of a vector bundle E over the unit ball. This operator maps to sections of the tensor product of E with the holomorphic tangent bundle. Let D be the conjugate operator. The higher-order Laplace operators $L_m = D^m \bar{D}^m$ are invariant differential operators under the group of biholomorphic mappings of the ball. In particular, L_1 is the negative of the invariant Laplace–Beltrami operator. The unit ball is a rank-1 Hermitian symmetric space and, in the case when E is a line bundle, all L_m are polynomials of L_1; see for example [Sha] for a general study on invariant differential operators on line bundles (in our case, everything follows from direct calculation). Our first result is an explicit formula for these polynomials. This is done with the help of the spherical transform on the line bundle studied in [Z]. For line bundles over the unit disk, the polynomials were found in [PZ]; in [EP] they were given for the trivial line bundle over the unit ball.

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