The Codimension-1 Property in Bergman Spaces over Planar Regions

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1. Introduction

Let G be a bounded planar region containing the origin in the complex plane $\mathbb{C}$. For $1 \leq p < \infty$, the Bergman space $L^p_a(G)$ consists of all analytic functions f in G with

$$\|f\|_p = \left(\int_G |f(z)|^p \, dA(z) \right)^{1/p} < \infty,$$

where dA denotes the Lebesgue measure on the complex plane.

Let ϕ be a smooth function with compact support. The Vitushkin localization operator T_ϕ is defined by

$$T_\phi f(z) = \int \frac{f(w) - f(z)}{w - z} \, \bar{\partial}\phi \, dA(w),$$

where f is a bounded function with compact support. Let $L^p(G)$ be the space of measurable functions that are zero off G, and let

$$\|f\|_p = \left(\int_G |f(z)|^p \, dA(z) \right)^{1/p} < \infty.$$

The Bergman space $L^p_a(G)$ is a closed subspace of the Banach space $L^p(G)$. It is well known that the operator T_ϕ is a bounded linear operator on $L^p(G)$ and leaves $L^p_a(G)$ invariant.

Let $H^\infty(G)$ denote the Banach algebra generated by bounded analytic functions on G. A closed subspace M of $L^p_a(G)$ is an $H^\infty(G)$ *invariant subspace* if it is invariant under multiplication by each bounded analytic function on G. The dimension of M/zM is no less than 1 since zero is in G. An $H^\infty(G)$ invariant subspace M satisfies the *codimension-1 property* if the dimension of M/zM is 1. Let $Z(M)$ be the set of common zeros of functions in M. We say that M has the *division property* if $f(z)/(z-\lambda)$ is in M whenever $\lambda \in G \setminus Z(M)$ and $f \in M$ with $f(\lambda) = 0$. In [5] it was shown that the codimension-1 property is actually equivalent to the division property. For $f_1, f_2, \ldots, f_n$ in $L^p_a(G)$, let $[f_1, f_2, \ldots, f_n]$ denote the

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