Fibers over the Sphere of a Uniformly Convex Banach Space

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1. Introduction: Bounded Analytic Functions on the Unit Ball

Over the past years, a significant interest has developed in the study of holomorphic functions defined on a domain in an infinite-dimensional Banach space and of their constituents (via Taylor expansions), the homogeneous polynomials. Many of the questions that have been studied have arisen from considerations of infinite-dimensional topology and from standard function algebra questions [CCG]. Recently, there has been an interest in connecting the well-developed theory of the geometry of Banach spaces with the function theory questions that have been studied classically, and some progress has been made in this direction [ACG; D; F; CCG; CGJ]. In addition, connections between properties of polynomials and geometry of the unit ball has been of interest (see [GJL] for a survey of this topic).

The present work is an attempt to study some of the properties of bounded analytic functions on the unit ball of an infinite-dimensional Banach space. In particular, we are interested in understanding something of boundary behavior; we combine techniques from the several fields to investigate it, especially with regard to the interplay with convexity and smoothness.

Many of the results here apply to the classical "nice" reflexive spaces, such as l_p and L_p $(1 < p < \infty)$. It is almost certain that there is much more to be learned even about the Hilbert space case.

We consider the boundary behavior of H^∞ functions on B, the open unit ball of an infinite-dimensional complex Banach space that has the geometric properties of uniform convexity, uniform smoothness, or both. By uniform smoothness, we mean uniform (real) Frechet differentiability of the norm, with the space considered as a real Banach space. Uniform convexity will mean that the dual is uniformly smooth; since spaces with either property are reflexive, this definition is complete. To be specific however, we state the following (after [LT]).

Definition 1.1. A complex Banach space is said to be *uniformly convex* (u.c.) if

$$\delta(\varepsilon) = \inf\left\{1 - \frac{\|x+y\|}{2} \;\middle|\; x, y \in X,\ \|x\| = \|y\| = 1,\ \|x-y\| = \varepsilon\right\} > 0 \quad \forall \varepsilon > 0.$$

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