Besov Spaces and Outer Functions

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1. Introduction

Let $\mathbb{D}$ denote the unit disk $\{z \in \mathbb{C} : |z| < 1\}$, $\mathbb{T}$ its boundary, and m the normalized Lebesgue measure on $\mathbb{T}$. For a function $f \in L^p$ $(:= L^p(\mathbb{T}, m))$, we define its L^p modulus of continuity by

$$\omega_p(t, f) := \sup_{-t \le h \le t} \left(\int_{\mathbb{T}} |f(e^{ih}\zeta) - f(\zeta)|^p \, dm(\zeta) \right)^{1/p}$$

for $0 \le t \le \pi$, and by

$$\omega_p(t, f) := \omega_p(\pi, f) \quad \text{for } \pi < t < \infty.$$

Further, given $0 < s < 1$, $0 < p < \infty$, and $0 < q < \infty$, the *Besov space* $B^s_{pq} = B^s_{pq}(\mathbb{T})$ is introduced as follows:

$$B^s_{pq} := \left\{ f \in L^p : \int_0^\infty \frac{\omega_p(t, f)^q}{t^{sq+1}} \, dt < \infty \right\}.$$

We shall mainly be concerned with the *analytic subspace*

$$AB^s_{pq} := B^s_{pq} \cap H^p,$$

where H^p is the classical Hardy space in the disk (see [9, Chap. II]). Alternatively, the class AB^s_{pq} can be described [15; 17] as the set of all analytic functions f on $\mathbb{D}$ satisfying

$$\int_0^1 (1 - r)^{(1-s)q-1} \left(\int_{\mathbb{T}} |f'(r\zeta)|^p \, dm(\zeta) \right)^{q/p} dr < \infty. \tag{1.1}$$

We remark that there is also a natural way to define the spaces B^s_{pq} and AB^s_{pq} with $s \ge 1$, but these are not considered in the present paper.

The problem we treat here is to characterize (the boundary values of) the moduli of functions in AB^s_{pq}. Thus, we consider a nonnegative function $\varphi \in L^p$ with

$$\int_{\mathbb{T}} \log \varphi \, dm > -\infty \tag{1.2}$$

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