Isomorphic Classification of the Spaces of Whitney Functions

Alexander P. Goncharov &
Mefharet Kocatepe

I. Introduction

Let $K \subset \mathbb{R}$ be a compact set such that $K = \overline{\text{int}\, K}$. By $\mathcal{E}(K)$ we denote the space of infinitely differentiable Whitney functions on K. This is the space of functions $f: K \to \mathbb{R}$ extendable to C^∞-functions on $\mathbb{R}$ equipped with the topology defined by the sequence of norms

$$\|f\|_q = |f|_q + \sup\{\,|(R_y^q f)^{(i)}(x)| \cdot |x-y|^{i-q} : \\ x, y \in K,\ x \neq y,\ i \leq q\,\},\ q = 0, 1, \dots,$$

where $|f|_q = \sup\{\,|f^{(j)}(x)| : x \in K,\ j \leq q\,\}$ and

$$R_y^q f(x) = f(x) - T_y^q f(x) = f(x) - \sum_{k=0}^{q} \frac{f^{(k)}(y)}{k!}(x-y)^k$$

is the Taylor remainder. With

$$U_q = \{\, f \in \mathcal{E}(K) : \|f\|_q \leq 1\,\},$$

the sequence (U_q) need not decrease, but the sets εU_q with $\varepsilon > 0$ and $q \in \mathbb{N}$ constitute a basis of neighborhoods of zero in $\mathcal{E}(K)$. It was shown in [20] by Tidten and in [25] by Vogt that the space $\mathcal{E}(K)$ is isomorphic to the space

$$s = \left\{ x = (\xi_n) : \|x\|_q = \sum_{n=1}^{\infty} |\xi_n| n^q < \infty\ \forall q \right\}$$

of rapidly decreasing sequences if and only if there is a continuous extension operator $L: \mathcal{E}(K) \to C^\infty(\mathbb{R})$.

Let $\mathbb{N} = \{1, 2, \dots\}$ and $\mathbb{Z}^+ = \{0, 1, 2, \dots\}$. We consider compact sets of the following type. For two sequences (a_n), (b_n) such that $0 < \cdots < b_{n+1} < a_n < b_n < \cdots < b_1 < 1$, let $I_n = [a_n, b_n]$ and $K = \{0\} \cup \bigcup_{n=1}^{\infty} I_n$. By ψ_n we denote the length of I_n; $h_n = a_n - b_{n+1}$ is the distance between I_n and I_{n+1}. In what follows we restrict ourselves to the case

$$\psi_n \searrow 0,\quad h_n \searrow 0,\quad \psi_n \leq h_n,\qquad n \in \mathbb{N}, \tag{1}$$

$$\exists Q \in \mathbb{N} : h_n \geq b_{n+1}^{Q},\qquad n \in \mathbb{N}. \tag{2}$$

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