A Theorem on Improving Regularity of Minimizing Sequences by Reverse Hölder Inequalities

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1. Introduction

The use of reverse Hölder inequalities pioneered by Gehring's celebrated lemma [5] in the theory of quasiconformal mappings has been well adapted in the calculus of variations for obtaining regularity of minimizers of integral functionals with certain natural growth conditions [6]. In this paper we elaborate upon some ideas of our recent paper [16] to prove a theorem on improving regularity of minimizing sequences of a family of integral functionals that do not satisfy the usual growth conditions but satisfy instead a uniform integral coercivity condition as given by (1.4) below. As an important application, we also prove a stability result on the strong convergence of the so-called *weakly almost conformal mappings* in $W^{1,p}(\Omega; \mathbf{R}^n)$ for certain p below the dimension n. See also [4; 7; 11; 13; 14; 16].

We begin with some notation. Let $\mathcal{M}^{n\times m}$ be the space of all real $n \times m$-matrices with norm $|X|$ defined by $|X|^2 = \operatorname{tr}(X^T X)$. For $p \geq 1$ and a domain D in $\mathbf{R}^m$, let $W^{1,p}(D; \mathbf{R}^n)$ be the usual Sobolev space of L^p-integrable maps $u: D \to \mathbf{R}^n$ having L^p-integrable gradients $(\nabla u)_{ij} = \partial u^i/\partial x_j$ for $1 \leq i \leq n$ and $1 \leq j \leq m$.

Let $\mathcal{K}$ be a closed subset of $\mathcal{M}^{n\times m}$, and let $d_{\mathcal{K}}(X) = \inf_{A\in\mathcal{K}}|X - A|$ be the distance function to $\mathcal{K}$. In this paper, we shall always assume that $d_{\mathcal{K}}$ satisfies the following condition:

$$d_{\mathcal{K}}(\lambda X) \leq K_0(d_{\mathcal{K}}(X) + 1), \quad X \in \mathcal{M}^{n\times m}, \ 0 \leq \lambda \leq 1. \tag{1.1}$$

Note that condition (1.1) is satisfied if $\mathcal{K}$ is a cone or a bounded set.

We consider the integral functionals $I_p(u; D)$ defined by

$$I_p(u; D) = \int_D d_{\mathcal{K}}^p(\nabla u(x))\, dx. \tag{1.2}$$

The natural admissible space for $I_p(u; D)$ is $W^{1,p}(D; \mathbf{R}^n)$, but we shall often consider $I_p(u; D)$ for all $u \in W^{1,1}_{\mathrm{loc}}(D; \mathbf{R}^n)$.

Throughout this paper, we assume that $1 \leq \alpha \leq \beta < \infty$ are given numbers, that $\Omega \subset\subset D_0$ are bounded smooth domains in $\mathbf{R}^m$, and that u_0 is a given map in $W^{1,\alpha}_{\mathrm{loc}}(D_0; \mathbf{R}^n)$ satisfying

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