The Bergman Projection and Vector-Valued Hardy Spaces

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1. Introduction and Statement of Results

Let S^n be the unit sphere in C^n, and let $d\sigma$ denote the surface area form on S^n normalized so $\int_{S^n} d\sigma = 1$; B_n will denote the unit ball and $d\nu$ will be the normalized volume form on B_n. We assume familiarity with the invariant Poisson integral and nonisotropic metric $d(\zeta, \eta) = |1 - \langle \zeta, \eta \rangle|^{1/2}$ used in the study of function theory on S^n; see [R, Chap. 5]. For $0 < p < \infty$, $H^p(S^n)$ is the usual space of distributions whose invariant Poisson integrals are holomorphic on B_n and whose admissible maximal functions belong to $L^p(d\sigma)$; see [R, Chap. 4]. For a function u defined on B_n and $1 < q < \infty$, let $A_q[u]$ be the *area function*

$$A_q[u](\zeta) = \left(\int_{\Gamma(\zeta)} |u(z)|^q \frac{d\nu(z)}{(1 - |z|)^{n+1}} \right)^{1/q},$$

where $\zeta \in S^n$ and $\Gamma(\zeta)$ is the usual approach region

$$\Gamma(\zeta) = \{ z \in B_n : |1 - \langle z, \zeta \rangle| < 1 - |z|^2 \}.$$

For $0 < p < \infty$, the *tent space* $T_q^p(B_n)$ consists of all functions u such that

$$\|u\|_{T_q^p} = \left(\int_{S^n} A[u]^p \, d\sigma \right)^{1/p} < \infty.$$

The tent space $T_q^\infty(B_n)$ consists of those functions u such that $|u(z)|^q \, d\nu(z)/(1 - |z|)$ is a Carleson measure; see [CMS].

It is well known that if $0 < p < \infty$ then a distribution F whose invariant Poisson integral is holomorphic belongs to $H^p(S^n)$ if and only if $u(z) = (1 - |z|) \times (|F(z)| + |\nabla F(z)|)$ belongs to T_2^p; here $F(z)$ is used to denote the invariant Poisson integral of F evaluated at $z \in B_n$. Another characterization of H^p for $0 < p < \infty$ is given in terms of the "g" function. If u is defined on B_n and $1 < q < \infty$, let

$$g_q[u](\zeta) = \left(\int_0^1 |u(t\zeta)|^q \frac{dt}{1 - t} \right)^{1/q}.$$

Then a distribution F with holomorphic Poisson integral belongs to H^p if and only if $g_2(u) \in L^p(d\sigma)$, where again $u(z) = (1 - |z|)(|F(z)| + |\nabla F(z)|)$; see [AB]. In each of these characterizations we have norm equivalences:

Received August 20, 1996. Revision received March 14, 1997.
Michigan Math. J. 44 (1997).