Equivariant Chern Character for the Invariant Dirac Operator

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1. Introduction

The attributes of elliptic pseudodifferential systems that have relevance within Atiyah–Singer index theory have been abstracted to the concept of Fredholm modules, or K-cycles in the language of K-homology. Pairing of a K-cycle with a K-cocycle gives the index. Connes invented cyclic cohomology while looking for a homology–cohomology formula of this pairing. The Chern–Connes character relates finitely summable Fredholm modules to cyclic cocycles and, more generally, relates Θ-summable Fredholm modules to entire cyclic cocycles. We refer to [5], and to [4] for the background and definitions.

The prototype for Θ-summable Fredholm modules is the Dirac K-cycle $(\mathcal{A}, \mathcal{H}, \mathrm{D})$, where $\mathcal{A} = C^\infty(M)$ for an even-dimensional Spin-manifold M, $\mathcal{H}$ is the graded Hilbert space of L^2 sections of the spinor bundle, and D is the Dirac operator, unbounded on $\mathcal{H}$, which is Θ-summable by Weyl's asymptotics for heat kernels. The Chern character is represented by the JLO-cocycle in the entire cyclic cohomology of $\mathcal{A}$. The Chern character has been computed in [2] by using symbol calculus [8] since $e^{-t^2\mathrm{D}^2}$ is a Getzler's asymptotic operator for which the leading symbol is known [2; 8; 16; 15]. In this paper we discuss the equivariant case; if G is a connected compact Lie group acting on M by isometries, then G-action commutes with Clifford multiplication and D is G-invariant, so we have an equivariant Θ-summable Fredholm module over the G-algebra $\mathcal{A}$. The corresponding equivariant Chern character in the equivariant entire cyclic cohomology (JLO-version) was defined in [10] and [13]. Apparently Getzler's symbol calculus does not apply directly here since $e^{-t^2\mathrm{D}^2} \cdot T$ is not an asymptotic operator in the sense of [2] for an orientation-preserving isometry T on M.

On the other hand, for classical geometric operators, direct heat-kernel asymptotic techniques were applied to the Hodge–de Rham and Riemann–Roch operators by Patodi and later improved by Yu [17] into a more essential form, called Clifford asymptotics for heat kernels. Using this technique, we are able to derive a cyclic cohomological formula for the equivariant Chern character for the equivariant entire cyclic cohomology.

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