Characterization of Convex Domains with Noncompact Automorphism Group

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Introduction

The conformal mapping theorem of Riemann asserts that a simply connected domain in $\mathbb{C}$, different from $\mathbb{C}$, is biholomorphically equivalent to the open unit disc $U = \{ z \in \mathbb{C} : |z| < 1 \}$. Many authors have been interested in the generalization of this result in several complex variables (cf. [2; 3; 4; 9; 14]). The situation is quite different there: a small C^2 perturbation of the unit ball $\mathbb{B}^{n+1}$ in $\mathbb{C}^{n+1}$ can be nonequivalent to $\mathbb{B}^{n+1}$, even if it is simply connected. This shows that a domain in $\mathbb{C}^{n+1}$ is not completely described by its topological properties. Thus one must study the automorphism group of a domain to find a polynomial representation of it, that is, a rigid polynomial domain and a biholomorphic equivalence between our original domain and this rigid polynomial domain.

From now on, we consider pseudoconvex domains with noncompact automorphism group. More precisely, we assume that there exists a family $(h_\nu)_\nu$ of automorphisms of Ω, a point p in Ω, and a point p_∞ in $\partial\Omega$ such that

$$\lim_{\nu \to \infty} h_\nu(p) = p_\infty.$$

We say that p_∞ is an *accumulating point* for an orbit of $\mathrm{Aut}(\Omega)$.

A very useful tool for constructing a biholomorphism from Ω to a rigid polynomial model domain is the scaling method introduced by Pinchuk [13]. We will describe the scaling for our problem in Section 2. This method allowed Bedford and Pinchuk [2] to prove that if a domain is bounded in $\mathbb{C}^2$, pseudoconvex, and real analytic of finite type $2m$ in the sense of d'Angelo [6] with noncompact automorphism group, then it is biholomorphic to the ellipsoid $E_m = \{ (z_1, z_2) \in \mathbb{C}^2 : |z_1|^2 + |z_2|^{2m} < 1 \}$, where $m \geq 1$. The scaling gives moreover a nice short proof of the Wong–Rosay theorem [14] when p_∞ is a point of strict pseudoconvexity.

Bedford and Pinchuk [3] used this method and an analysis of vector fields to prove that if a domain Ω is bounded in $\mathbb{C}^{n+1}$, smooth, convex, and of finite type, and if $\mathrm{Aut}(\Omega)$ is noncompact, then Ω is biholomorphically equivalent to a weighted homogeneous convex rigid polynomial domain $D = \{ (z_0, z') \in \mathbb{C} \times \mathbb{C}^n : \mathrm{Re}\, z_0 + P(z') < 0 \}$. In the bounded case, the noncompactness of the automorphism group is equivalent to the existence of an accumulating point, and it seems relevant that the domain could be characterized by its geometry near this

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