Stability for a Class of Foliations Covered by a Product

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Introduction

In this paper, we study transversely orientable, codimension-1 C^1 foliations of Riemannian 3-manifolds. In particular, we examine foliations of a manifold M that are covered by the canonical foliation of $\mathfrak{R}^3$ by parallel hyperplanes, which we refer to as "covered by a product". These foliations are particularly nice in the sense that they are completely determined by the induced action of $\pi_1(M)$ on the real line, which is the leaf space of the universal cover (see [20]). This family of foliations includes fibrations as well as weak stable (or unstable) foliations associated with many Anosov flows, including geodesic flows or suspensions of Anosov diffeomorphisms. Several recent works have focused on whether the associated foliations of other Anosov flows are covered by a product (e.g., [1; 3; 5]).

For closed M, foliations that are covered by a product constitute a large subset of the taut foliations. However, they are strictly a proper subset, as shown by an example in Section 2. While the property of being taut is stable for closed manifolds in the sense that all C^1 close foliations are also taut [21], this is not at all clear for the property of being covered by a product. (We refer to the metric on the space of C^1 foliations, defined by Hirsch in [10]). In this paper, we consider the class of foliations of a closed manifold $M \neq S^2 \times S^1$ that have a transverse loop which lifts to a copy of the leaf space in the universal cover and hence are covered by a product. We find conditions that are sufficient to ensure these foliations are stable in the sense that nearby foliations are also in this class.

More precisely, we find a condition on a branched surface W constructed from a foliation of a manifold M that is sufficient to guarantee the existence of such a transverse loop τ. Under certain conditions (given in Lemma 2.3), the branched surface W can then be modified to obtain a branched surface W' with the property that, for every foliation carried by W' and covered by a product, τ is a transverse loop that is covered by a copy of the leaf space. We denote by $\hat{W}'$ the lift of W' to the universal cover of M. The covering of τ is a curve that is transverse to $\hat{W}'$, and if it intersects a set of smooth submanifolds in a particular manner (which we make explicit at the end of Section 1) then we call it a *global transversal* for $\hat{W}'$. In this case we have that, for every foliation carried by W', τ is a transverse loop that is covered by a copy of the leaf space. In short, we show the following.

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