Controls on the Plus Construction

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0. Introduction

Given a closed n-manifold M ($n \geq 5$), a finitely presented group G, and an epimorphism $\mu\colon \pi_1(M) \to G$ with perfect kernel, Quillen's plus construction [Q] provides a cobordism (W, M, N) where W is a compact manifold satisfying:

(1) $\partial W = M \cup N$;
(2) the inclusion $N \to W$ is a homotopy equivalence; and
(3) $\pi_1(N)$ is isomorphic to the quotient G $(\cong \pi_1(M)/\ker(\mu))$.

We obtain a more controlled version of this construction, namely, a closed map $p\colon W \to [-1, 1]$ satisfying the additional properties:

(4) $p^{-1}(t)$ is a manifold for each $t \in [-1, 1]$; and
(5) $M = p^{-1}(1)$ and $N = p^{-1}(-1)$.

We call a map $p\colon W \to [-1, 1]$ satisfying properties (4)–(5) a *crumpled lamination on* (W, M, N) and denote it by the 4-tuple (W, M, N, p).

Our earlier constructions required special hypotheses on $\ker(\mu)$ in order to construct crumpled laminations. Here, we show none are needed. In particular, our following main result asserts that any cobordism arising from a Quillen plus construction admits a crumpled lamination.

MAIN THEOREM (THEOREM 1.1). *For any closed n-manifold M* ($n \geq 6$), *finitely presented group G, and epimorphism* $\mu\colon \pi_1(M) \to G$ *with perfect kernel, there is a cobordism W admitting a crumpled lamination* (W, M, N, p), *where* $p^{-1}([-1, 0]) \approx N \times [-1, 0]$, $p^{-1}((0, 1]) \approx M \times (0, 1]$, $\pi_1(N) \cong G$, *the inclusion* $N \to W$ *is a homotopy equivalence, and the inclusion* $M \to W$ *induces the homomorphism* μ *on fundamental groups.*

Simple homotopy theory and prior results [DT2] then yield the following characterization.

MAIN COROLLARY (COROLLARY 1.3). *Let* (W, M, N) *be a compact cobordism with* $n = \dim(M) \geq 6$. *Then W admits a crumpled lamination*

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