The Backward Shift on Weighted Bergman Spaces

ALEXANDRU ALEMAN & WILLIAM T. ROSS

1. Introduction

Let $H^2(\mathbb{D}) = H^2$ denote the *Hardy space* of analytic functions $f = \sum a_n z^n$ on the open unit disk $\mathbb{D} = \{|z| < 1\}$ for which

$$\sup_{0<r<1} \int_{|\zeta|=1} |f(r\zeta)|^2 \frac{|d\zeta|}{2\pi} = \sum_{n=0}^{\infty} |a_n|^2 < +\infty.$$

It is known that the *backward shift operator*

$$Lf = \frac{f - f(0)}{z}$$

is continuous on H^2 and the subspaces (closed linear manifolds) $\mathfrak{M} \subset H^2$ for which

$$L\mathfrak{M} \subset \mathfrak{M}$$

(such $\mathfrak{M}$ will be called *L-invariant* or *backward shift-invariant subspaces*) were completely characterized in [8] by means of duality.

NOTATION. We pause here to set some important notation that will be used throughout the paper. If $\mathcal{B}$ is a Banach space and T is a bounded linear operator on $\mathcal{B}$, we let $\operatorname{Lat}(T, \mathcal{B})$ denote the subspaces $\mathfrak{M} \subset \mathcal{B}$ for which $T\mathfrak{M} \subset \mathfrak{M}$. For a set $S \subset \mathcal{B}$, we let $[S]_{(T,\mathcal{B})}$ denote the smallest T-invariant subspace of $\mathcal{B}$ that contains the set S. In this case, we will say $[S]_{(T,\mathcal{B})}$ is the T-invariant subspace "generated" by S.

The dual of H^2 can be identified with H^2 by means of the pairing

$$\langle f, g \rangle = \lim_{r \to 1^-} \int_{|\zeta|=1} f(r\zeta)\overline{g(r\zeta)} \frac{|d\zeta|}{2\pi}, \tag{1.1}$$

and a simple computation with power series reveals

$$\langle Lf, g \rangle = \langle f, zg \rangle \quad \forall f, g \in H^2.$$

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