Convolutions and Fourier–Feynman Transforms of Functionals Involving Multiple Integrals

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1. Introduction

Let $C_0[0,T]$ denote one-parameter Wiener space, that is, the space of $\mathbb{R}$-valued continuous functions $x(t)$ on $[0,T]$ with $x(0)=0$. The concept of an L_1-analytic Fourier–Feynman transform for functionals on Wiener space was introduced by Brue in [1]. In [3], Cameron and Storvick introduced an L_2-analytic Fourier–Feynman transform. In [11], Johnson and Skoug developed an L_p-analytic Fourier–Feynman transform for $1 \le p \le 2$ that extended the results in [1; 3]. In [9], Huffman, Park, and Skoug defined a convolution product for functionals on Wiener space and, for a class of functionals of the type

$$F(x)=f\left(\int_0^T \alpha_1(t)\,dx(t),\dots,\int_0^T \alpha_n(t)\,dx(t)\right),$$

showed that the Fourier–Feynman transform of the convolution product was a product of Fourier–Feynman transforms. In [10], they obtain similar results for functionals of the form

$$G(x)=\exp\left\{\int_0^T g(t,x(t))\,dt\right\},$$

which play an important role in quantum mechanics.

In this paper we consider functionals, on Wiener space, of the form

$$F(x)=\exp\left\{\int_0^T\int_0^T f(s,t,x(s),x(t))\,ds\,dt\right\} \tag{1.1}$$

for appropriate $f\colon [0,T]^2\times\mathbb{R}^2\to\mathbb{C}$. Such functionals were discussed in the book by Feynman and Hibbs [8, Secs. 3-10] on path integrals, and in Feynman's original paper [7, Sec. 13]. Feynman obtained such functionals by formally integrating out the oscillator coordinates in a system involving a harmonic oscillator interacting with a particle moving in a potential. The double dependence on time occurs because, as Feynman and Hibbs [8, p. 71] explain, "The separation of past and future can no longer be made. This

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