Local Homology Properties of Boundaries of Groups

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0. Introduction

In this paper we formalize the concept of the boundary of a group (Definition 1.1). Even though a group might have different boundaries, certain global and local homological invariants of the boundary are determined by the cohomological invariants of the group. In particular, we show (Theorem 2.8) that the boundary of a Poincaré duality group is a homology manifold.

1. Boundaries of Groups

1.1. $\mathcal{Z}$-Structures on Groups

Recall that a compact metrizable space is a *Euclidean retract* (or ER) if it can be embedded in some Euclidean space as its retract, or (equivalently) if it is finite-dimensional, contractible, and locally contractible. A closed subset Z of a Euclidean retract $\tilde{X}$ is said to be a *Z-set* if any of the following equivalent conditions hold.

(a) There is a deformation $h_t: \tilde{X} \to \tilde{X}$ with $h_0 = \mathrm{id}$ and $h_t(\tilde{X}) \cap Z = \emptyset$ for $t > 0$.
(b) For every $\epsilon > 0$ there is a map $f: \tilde{X} \to \tilde{X}$ that is ϵ-close to the identity and whose image misses Z.
(c) For every open set $U \subset \tilde{X}$, the inclusion $U \setminus Z \hookrightarrow U$ is a homotopy equivalence.

Definition 1.1. Let G be a group. A *$\mathcal{Z}$-structure* on G is a pair $(\tilde{X}, Z)$ of spaces satisfying the following four axioms.

(1) $\tilde{X}$ is an ER.
(2) Z is a Z-set in $\tilde{X}$.
(3) $X = \tilde{X} \setminus Z$ admits a covering space action of G with compact quotient.

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