Inductive Limits of Algebras of Generalized Analytic Functions

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We study inductive limits of algebras of generalized analytic functions generated by compact abelian groups with ordered duals. In particular, we answer a question raised in [2] for inductive limits of spaces of type H^∞ on a compact abelian group with ordered dual.

1. Introduction

Let Γ be a subgroup of the group $\mathbb{R}$ of real numbers. We assume that Γ is equipped with the discrete topology. Denote by G the dual group of Γ, that is, G is the group of characters of Γ. Note that G is a compact abelian group with unit e.

In what follows we make use of the terminology and notation from [7]. By the Pontryagin duality theorem, the dual group $\hat{G}$ of G is isomorphic to Γ. For a given $a \in \Gamma$ let $\chi^a \in \hat{G}$ be the character $\chi^a(g) = g(a)$, $g \in G$. Let σ be the normalized Haar measure on G. Every function f in $L^1(G, \sigma)$ relative to σ has a formal Fourier series

$$f(g) \sim \sum_{a \in \Gamma} c_a^f \chi^a(g),$$

where

$$c_a^f = \int_G f(g) \bar{\chi}^a(g)\, d\sigma(g)$$

are the Fourier coefficients of f. The set $S(f)$ of numbers a in Γ for which $c_a^f \neq 0$ is the *spectrum* of f. A function $f \in L^1(G, \sigma)$ is called a *generalized analytic function on G* if $S(f)$ is contained in the semigroup $\Gamma_+ = \{a \in \Gamma \mid a \geq 0\}$.

Let Δ_G be the set of semi-characters (i.e., homomorphisms from Γ_+ into the unit disc in $\mathbb{C}$) of the semigroup Γ_+. Δ_G is called the *big disc* over G. It is well known that Δ_G is a compact set and can be obtained from the Cartesian product $[0,1] \times G$ by identifying the points in the fiber $\{0\} \times G$. Every point $m \in \Delta_G$ can be expressed in the *polar* form $m = rg$ for some $r \in [0,1]$ and $g \in G$. Observe that $G \equiv \{1\} \times G \subset \Delta_G$ since the characters on Γ are semi-

Received July 7, 1995.

Research of the second author was partially supported by a grant from the National Science Foundation.

Michigan Math. J. 42 (1995).