The Pluri-Complex Green Function and a Covering Mapping

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1. Introduction

Myrberg [12] proved that, if M is a hyperbolic Riemann surface with the Green function $g^M(\cdot, p)$ with pole at $p \in M$ and if $\pi: E \to M$ is a covering mapping from the unit disk $E = \{\lambda \in \mathbf{C}; |\lambda| < 1\}$ in $\mathbf{C}$ to M, then

$$g^M(q, p) = \sum_{j \geq 0} \log \left| \frac{1 - \bar{a} b_j}{b_j - a} \right|,$$

where $a \in \pi^{-1}(p)$ and $\{b_0, b_1, \dots\} = \pi^{-1}(q)$.

In any complex manifold M, we can define the pluri-complex Green function $G_p^M(\cdot)$ with pole at $p \in M$ in such a manner that if M is a hyperbolic Riemann surface, the negative of $G_p^M(\cdot)$ is nothing other than the Green function on M with pole at p [9; 10; 11; 2; 3; 6; 8]. Since $b \mapsto \log|(1 - \bar{a}b)/(b - a)|$ is the Green function on E with pole at $a \in E$, Myrberg's theorem is rewritten as follows:

$$G_p^M(q) = \sum_{j \geq 0} G_a^E(b_j),$$

where $a \in \pi^{-1}(p)$ and $\{b_0, b_1, \dots\} = \pi^{-1}(q)$.

In this paper we shall show the following.

THEOREM A. *Let $\pi: N \to M$ be a covering mapping from a complex manifold N to another one M. For $p, q \in M$, let $a \in \pi^{-1}(p)$ and $\{b_0, b_1, \dots\} = \pi^{-1}(q)$. Then*

$$G_p^M(q) \geq \sum_{j \geq 0} G_a^N(b_j).$$

When the covering is regular, we have the following.

THEOREM B. *Let $\pi: N \to M$ be a regular covering mapping from a complex manifold N to another one M. For $p, q \in M$, let $\{a = a_0, a_1, \dots\} = \pi^{-1}(p)$ and $\{b = b_0, b_1, \dots\} = \pi^{-1}(q)$. Then*

$$G_p^M(q) \geq \sum_{j \geq 0} G_a^N(b_j) = \sum_{j \geq 0} G_{a_j}^N(b).$$

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