The Topology of Minimal Surfaces in Seifert Fiber Spaces

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1. Introduction

A basic question in the theory of minimal surfaces in 3-dimensional manifolds is to decide which embeddings of surfaces can be realized by minimal surfaces. Fundamental results were obtained in the case of Riemannian metrics of positive curvature in [Fr], [L1], and [SY] for sectional, Ricci, and scalar curvatures, respectively. In [R1] a fairly complete description was obtained of the topology of embeddings of minimal surfaces in 3-manifolds of positive scalar curvature.

Seifert fiber spaces are an important class of examples of 3-dimensional manifolds that admit 1-dimensional foliations by circles. Thurston [Th] has proposed a geometrization program for classifying closed 3-manifolds by decomposing them into pieces that admit eight geometries. Six of the eight geometries occur on Seifert fiber spaces. Moreover, the natural metrics are compatible with the Seifert fiber structure, in the sense that (possibly after passing to a double cover) the isometry group of the metric has an $SO(2)$ component with orbits the circle fibers.

In [Ha], Hass studied the topology of π_1-injective minimal surfaces in Seifert fiber spaces. In Section 3 we obtain a topological classification of arbitrary embedded minimal surfaces in such 3-manifolds, extending [Ha]. Finally in Section 4, using the minimax technique developed in [Pi], [PR1], [PR2], and [HPR], examples of interesting minimal surfaces in Seifert fiber spaces are constructed. Note that, throughout this paper, the only restriction on the Riemannian metric is that the $SO(2)$ action of the previous paragraph be by isometries.

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2. Seifert Fiber Spaces

For details about Seifert fiber spaces, a good reference is Orlik [Or].

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