Nonlinear Potential Theory on the Ball, with Applications to Exceptional and Boundary Interpolation Sets

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0. Introduction

Let S denote the boundary of B_n, the unit ball in $\mathbf{C}^n$, and let $d\sigma$ be the usual rotation-invariant measure defined on S. If $f \in L^1(d\sigma)$, $\eta \in S$, and $0 < \beta < n$, then we define the non-isotropic potential

$$I_\beta f(\eta) = \int_S \frac{f(\zeta)}{|1 - \langle \eta, \zeta \rangle|^{n-\beta}}\, d\sigma. \tag{0.1}$$

We also set

$$I_\beta \mu(\eta) = \int_S \frac{d\mu}{|1 - \langle \eta, \zeta \rangle|^{n-\beta}} \tag{0.2}$$

for any finite measure μ ($\mu \in \mathfrak{M}(S)$).

If $1 < p < \infty$, let L^p_β be the space of potentials $F = I_\beta f$ where $f \in L^p(d\sigma)$ with norm

$$\|F\|_{L^p_\beta} = \|f\|_{L^p}.$$

The space L^p_β is an analog of the usual potential space defined in Euclidean space. In the case where β is an integer, L^p_β coincides with a (non-isotropic) Sobolev space.

We will also need the Hardy–Sobolev spaces H^p_β ($0 < \beta, p < \infty$) of functions F holomorphic in the unit ball. Let

$$F(z) = \sum_k f_k(z) \tag{0.3}$$

be the homogeneous polynomial expansion of F (see [Ru]) and

$$R^\beta F(z) = \sum_k (1+k)^\beta f_k(z) \tag{0.4}$$

its (radial) fractional derivative of order β. Then H^p_β is the space of all holomorphic functions F on B_n with the property that

$$\|F\|_{H^p_\beta} = \sup_{0<r<1} \|R^\beta F(r\zeta)\|_{L^p(d\sigma)} < \infty. \tag{0.5}$$

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