The Topological Whitehead Torsion of an Equivariant Fiber Homotopy Equivalence

STRATOS PRASSIDIS

Introduction

In this paper we compute the topological equivariant torsion of an equivariant fiber homotopy equivalence between compact equivariant ANRs. Throughout this paper, G denotes a finite group. Let $p: E \to B$ and $p': E' \to B'$ be locally trivial G-fibrations between compact G-ANRs such that the fibers are equivariant compact ANRs. Let

$$E' \xrightarrow{h} E$$

$$p' \downarrow \qquad \downarrow p$$

$$B' \xrightarrow{f} B$$
(*)

be a G-fiber homotopy equivalence over the G-homotopy equivalence f. Then we compute the topological torsion of h using the torsion of the pullback of f and the fiberwise action of the equivariant Euler characteristic of B' to the torsion of the fibers (Theorem 6.5). This result generalizes the main theorems in [1], [2], and [7].

The structure of the paper is as follows: In Section 1 we summarize the properties of compact G-ANRs and G-CE maps between them. In Section 2 we recall the definition of the topological torsion of a G-homotopy equivalence between compact G-ANRs and we prove the composition and the sum formula. In Section 3 we summarize the theory of functorial additive invariants [6, Chap. IV; 11, §6]. As an application of this theory we prove the product formula for the topological torsion following the lines of proof of Theorem 6.11 in [11]. This product formula generalizes the one given in the cellular case in [8] and [13]. In Section 4 we define the pull-back map p^* : $\operatorname{Wh}_G^{\operatorname{Top}}(B) \to \operatorname{Wh}_G^{\operatorname{Top}}(E)$ determined by a locally trivial G-fibration $p: E \to B$ between compact G-ANRs. The definition and the properties of p^* are analogous to the ones proven in [2] for G = 1. In Section 5 we define the fiberwise product of the Euler characteristic of B' with the torsions of the homotopy equivalences of the fibers determined by (*). In Section 6 we complete the computation of the topological torsion of h. We first give the proof for the special case that B is a finite G-complex. Using the fact that every compact