Invariant Subspaces in VMOA and BMOA

DINESH SINGH & U. N. SINGH

1. Introduction

In this article we describe completely the invariant subspaces of the operator S of multiplication by the coordinate function z on the Banach spaces BMOA and VMOA. BMOA stands for analytic functions of bounded mean oscillation on the unit circle T and VMOA is its subspace consisting of functions of vanishing mean oscillation. Due to the nonseparability of BMOA, the nature of the invariant subspaces is somewhat like their nature in the case of H^∞. Just as in the case of H^∞, we describe those invariant subspaces of BMOA that are closed in the weak-star topology when BMOA is treated as the dual of the Hardy space H^1. On VMOA, the invariant subspaces that are characterized are closed in the norm topology. Inner functions play a central role in both cases. For precise statements we refer to Section 2, Theorem A and Theorem C.

We also prove that the maximal ideals of the Banach algebra QA which correspond to fibers are dense in VMOA. The same kind of density result holds in the weak-star topology of BMOA for the maximal ideals of H^∞ that correspond to fibers. See Section 2, Theorem B and Theorem D. Quite interestingly, outer functions in both spaces turn out to be cyclic vectors for the operator S. For BMOA the cyclicity is in the context of the weak-star topology.

In the remainder of this section we outline very briefly those parts of the theory of Hardy spaces and BMOA that will be needed in the rest of this paper. Section 2 contains the precise statements of the main results. Section 3 contains preliminary results. Sections 4, 5, 6, and 7 are the proofs of the main results, namely, Theorems A, B, C, and D (respectively).

T will denote the unit circle in the complex plane and D its interior. L^p and H^p will denote the familiar Lebesgue and Hardy spaces on T, with $\|f\|_p$ as the L^p norm of f. It is well-known that each element f of H^p can be looked upon as an analytic function in D satisfying a certain growth condition. Of great importance is the fact that any analytic $f(z)$ in H^p can be uniquely factorized as $f(z)=B(z)S(z)O(z)$, where $B(z)S(z)$ is the inner factor of $f(z)$ and $O(z)$ is its outer factor. For further details we refer to [3] or [5].

Received June 26, 1989. Revision received April 12, 1993.
Michigan Math. J. 41 (1994).