Removable Singularities for L^p CR Functions

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1. Introduction

Let Ω be a bounded domain in $\mathbf{C}^n$ with C^2-smooth boundary $b\Omega$. A closed subset X of $b\Omega$ is said to be *removable for continuous CR functions* if, for each function f continuous on $b\Omega\setminus X$ satisfying the tangential Cauchy–Riemann equations in the weak sense on $b\Omega\setminus X$, there exists a function F holomorphic in Ω continuously assuming the boundary values f on $b\Omega\setminus X$. There have been many interesting results proved recently relating removability to convexity with respect to various function spaces. For example, define the $O(\bar{\Omega})$-hull of $X\subset b\Omega$ to be the set $\hat{X}_\Omega$ of all points $p\in\bar{\Omega}$ such that $|\phi(p)|\leq \max\{|\phi(z)|: z\in X\}$ for all functions ϕ holomorphic in a neighborhood of $\bar{\Omega}$. The following result of Stout [10] will be important for us: Let Ω be a strictly pseudoconvex domain in $\mathbf{C}^n$, X a compact subset of $b\Omega$. If f is a continuous CR function on $b\Omega\setminus X$, then there exists a function holomorphic in $\Omega\setminus\hat{X}_\Omega$, continuous on $\bar{\Omega}\setminus\hat{X}_\Omega$, with $F=f$ on $b\Omega\setminus X$. In particular, Stout's theorem implies that if $X=\hat{X}_\Omega$ (we say X is *$O(\bar{\Omega})$-convex*) then X is removable. In $\mathbf{C}^2$, the converse is also true: If X is contained in the boundary of a strictly pseudoconvex domain and X is removable for continuous CR functions, then X is $O(\bar{\Omega})$-convex. Stout's paper [11] gives an excellent survey of results on removable singularities for CR functions.

We wish to study removable singularities for other classes of CR functions. Fix p, $1\leq p\leq\infty$, and let σ be the induced $(2n-1)$-dimensional Euclidean measure on $b\Omega$. Let us say that $X\subset b\Omega$ is *removable for L^p CR functions* if, for each $f\in L^p(b\Omega, d\sigma)$ satisfying the tangential Cauchy–Riemann equations on $b\Omega\setminus X$, there exists F in the Hardy space $H^p(\Omega)$ with boundary values f σ-almost everywhere on $b\Omega\setminus X$. In view of Stout's theorem above, it is reasonable to direct our attention first to $O(\bar{\Omega})$-convex subsets of $b\Omega$. Even in the simplest case, where $\Omega=B$ is the unit ball in $\mathbf{C}^n$ and $O(\bar{\Omega})$-convexity is equivalent to polynomial convexity, such sets can be quite large —there exist polynomially convex subsets of bB with positive σ-measure (see [11]). We shall restrict our attention to sets of $(2n-1)$-dimensional measure zero. On the other hand, if the Hausdorff dimension of X is sufficiently small, then the arguments of [11] for the case of L^∞ functions can be adapted

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