# A Factorization Theorem for Smooth Crossed Products

LARRY B. SCHWEITZER

## Introduction

By a remarkable theorem of Dixmier and Malliavin [DM, Thm. 3.3], it is known that the convolution algebra $C_c^\infty(G)$ of compactly supported $C^\infty$-functions on a Lie group $G$ satisfies the factorization property—namely, that every set of $C^\infty$-vectors $E$ for the action of $G$ is equal to the finite linear span $C_c^\infty(G)E$. In this paper, we replace $C_c^\infty(G)$ by the smooth crossed products for transformation groups $G \rtimes \mathcal{S}(M)$ defined in [S1]. We define an appropriate notion of a differentiable $G \rtimes \mathcal{S}(M)$-module, which generalizes the notion of $C^\infty$-vectors for actions of Lie groups. (This definition was first introduced by Du Cloux [D1; D2]). Under the assumption that the Schwartz functions $\mathcal{S}(M)$ vanish rapidly with respect to a continuous, proper map $\sigma: M \to [0, \infty)$, we then show that $G \rtimes \mathcal{S}(M)$ satisfies the factorization property—namely, that any differentiable $G \rtimes \mathcal{S}(M)$-module $E$ is the finite span of elements of the form $ae$, where $a \in G \rtimes \mathcal{S}(M)$ and $e \in E$. In the course of doing this, we also show that if a Fréchet algebra $A$ has the factorization property, then the smooth crossed product $G \rtimes A$ does also.

Other aspects of the representation theory of the smooth crossed products $G \rtimes \mathcal{S}(M)$ are studied in [S2]. I would like to thank Berndt Brenken for a pleasant stay at the University of Calgary, where I wrote the first draft of this paper.

## 1. Differentiable Representations and Multipliers

We define what it means for an algebra and a representation to be differentiable. We shall use representation and module terminology interchangeably throughout this paper. Everything we do will be for left modules, though similar statements are also true for right modules.

DEFINITION 1.1. By a *Fréchet algebra* we mean a Fréchet space with an algebra structure for which the multiplication is jointly continuous. (We do not assume that Fréchet algebras are $m$-convex.) Let $A$ be a Fréchet algebra. By a *Fréchet A-module* we mean a Fréchet space $E$ that is an $A$-module for

Received July 6, 1992. Revision received February 1, 1993.
Michigan Math. J. 41 (1994).