On the Dirichlet Problem for the Complex Monge–Ampère Operator

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1. Introduction

Let D be a bounded domain in $\mathbf{C}^n$. Given $f \in C(D)$ with $f \geq 0$ and given $\phi \in C(\partial D)$, we study the nonlinear Dirichlet problem:

$$\begin{aligned} &u \text{ is plurisubharmonic (psh) in } D, \text{ i.e., } u \in P(D), \\ &(dd^c u)^n = f^n \, dV \text{ in } D, \quad \text{and} \\ &u = \phi \text{ on } \partial D \end{aligned} \tag{1.1}$$

where $(dd^c(\cdot))^n$ is the complex Monge–Ampère operator studied extensively by Bedford and Taylor. For D strictly pseudoconvex, existence and uniqueness of the solution u were shown in [BT1]. The same result holds more generally for the class of B-regular domains introduced by Sibony [Si] (for the definition of B-regular, see Section 2). For further results when $f \in L^2(D)$ we refer the reader to [CP].

In Section 2 we outline an iterative balayage-type procedure for constructing u which uses only classical potential theory in R^{2n}. The idea is motivated by the fact that for u in $P(D) \cap C^2(D)$,

$$\left[\det\left(\frac{\partial^2 u}{\partial z_i \partial \bar{z}_j}\right)\right]^{1/n} = \frac{1}{n} \inf\{\Delta_a u : a \in A\},$$

where

$$A = \{a \in GL(n, \mathbf{C}) : a \text{ is positive definite and Hermitian with } \det a = 1\} \tag{1.2}$$

and

$$\Delta_a u = \sum a_{ij} \frac{\partial^2 u}{\partial z_i \partial \bar{z}_j} = a-\text{Laplacian of } u. \tag{1.3}$$

Our construction may be considered as a potential-theoretic interpretation of Gaveau's approach to (1.1) in [G1]. For a different approach to the homogeneous equation ($f \equiv 0$), see Poletsky [Po] and Bedford [Be]. We should also call attention to Bremermann's work [Br].

In Section 3 we study (1.1) for the bidisc $\mathbf{U}$ in $\mathbf{C}^2$. This domain is not B-regular. However, the homogeneous Monge–Ampère equation for $\mathbf{U}$ was

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