Embedding Theorems for Spaces of Analytic Functions via Khinchine's Inequality

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1. Introduction

We show how known embeddings of sequence spaces into spaces of analytic functions on the unit disk $\mathbf{D}$ (and other domains) can be combined with Khinchine's inequality to obtain lower estimates on expressions like

$$\int_{\mathbf{D}} |f^{(n)}|^s \, d\mu.$$

These lower estimates allow us to obtain necessary conditions on the measure μ on $\mathbf{D}$ in order that there exists a constant C with

$$\left(\int_{\mathbf{D}} |f^{(n)}|^s \, d\mu\right)^{1/s} \leq C\|f\|.$$

(Here $\|f\|$ denotes the norm appropriate for the space of analytic functions.) In each of the cases presented here, the necessary condition coincides with a sufficient condition obtained by straightforward estimates. Moreover, this necessary and sufficient condition reduces to the characterization of the multipliers between certain related sequence spaces. The spaces of analytic functions considered include the Bergman spaces, the mixed norm spaces, and a new class of spaces: the analytic functions belonging to certain weighted tent spaces. There is essentially no restriction on the exponent s and the exponents defining the spaces of analytic functions. Moreover, more general expressions than the $L^s(\mu)$ norm may appear on the left-hand side of the above inequality.

Let $\mathbf{D}$ be the open unit disk in the complex plane $\mathbf{C}$ and let A^p denote the Bergman space of analytic functions f on $\mathbf{D}$ whose L^p norm $\|f\|_p \stackrel{\text{def}}{=} (\int_{\mathbf{D}} |f|^p \, dA)^{1/p}$ is finite (dA is area measure). Let ρ denote the *pseudohyperbolic metric*

$$\rho(z,w) \stackrel{\text{def}}{=} \left|\frac{z-w}{1-\bar{z}w}\right|$$

on $\mathbf{D}$ and

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