Summation Conditions on Weights

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1. Introduction and Notation

In this paper, we introduce a class of summation conditions on weights which are equivalent to the dyadic weight conditions A_∞^d, A_p^d, and B_p^d, and provide a useful alternative way of thinking of these weight conditions. We then use this equivalence result to find a new proof of the boundedness of the dyadic square function on $L^p(w)$ for any A_p^d weight w. (Usually one shows, as in [4], that singular integrals, square functions, and related operators are bounded on weighted $L^p(w)$ spaces by using a good-λ inequality, but we avoid such methods entirely.)

Our first task (Section 2) is to state and prove the main equivalence theorem. The summation conditions we introduce here are related to the conditions introduced by R. Fefferman, Kenig, and Pipher in [6], but the methods employed are completely different. In Section 3, we utilize the results and ideas of Section 2 to prove the boundedness of the dyadic square function on weighted $L^p(w)$ spaces.

Harmonic analysis on "product spaces" has been the subject of much scrutiny in recent years (an overview of this field can be found in [3]), and so we finish, in Section 4, by defining analogs of our summation conditions on product spaces and by showing that they are related to the product A_p^d and B_p^d conditions.

Throughout this paper, we will use "C" to indicate a constant that depends only on p and the dimension n. $\mathfrak{D}=\mathfrak{D}(\mathbf{R}^n)$ indicates the set of all dyadic cubes in $\mathbf{R}^n$. For any $Q\in\mathfrak{D}$, $\mathfrak{D}(Q)$ is the collection of proper dyadic subcubes of Q, and $\tilde{Q}$ is the dyadic double of Q (the smallest dyadic cube properly containing Q). For any weight w and set S, $w(S)$ denotes the integral of w over S, $|S|$ denotes the Lebesgue measure of S, and $w_S=w(S)/|S|$. Unless otherwise specified, $1<p<\infty$, but p is otherwise arbitrary.

2. A_p^d, B_p^d, and Summation Conditions

In this section, we shall examine conditions on a weight w involving the sum

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