Propagation of Singularities in a Locally Integrable Structure

S. BERHANU

0. Introduction

In a recent paper [13] Trépreau proved several theorems about the propagation of singularities for *CR* functions defined on generic *CR* submanifolds of $\mathbf{C}^n$. This work extends some of his results to a general first-order system of PDEs for which the question of holomorphic extendability (more precisely, hypoanalyticity [1]) makes sense.

A few years ago, Hanges and Treves [5] proved that connected elliptic submanifolds of a hypoanalytic manifold Ω propagate hypoanalyticity of a solution. When Ω is a *CR* manifold, elliptic submanifolds coincide with complex submanifolds and "hypoanalyticity of a solution" means holomorphic extendability of a *CR* distribution. As a corollary of our result, we will derive a microlocal version of the Hanges–Treves theorem; that is, we will get a propagator of microlocal hypoanalyticity in the part of the cotangent space $T^*\Omega$ lying above an elliptic submanifold. This corollary will in turn imply the main result of [5].

Another corollary concerns the propagation of microlocal analyticity for solutions of a formally integrable system of real analytic vector fields. It is shown here that the propagation occurs along a Nagano leaf [8] generated by the Hamiltonians of the real and imaginary parts of the vector fields and contained in the characteristic set of the system. We mention that Hanges and Sjöstrand [4] proved such a propagation for solutions of a differential operator of principal type with real analytic coefficients. In the case of systems of analytic vector fields, we believe the approach here is simpler. This paper is also related to the work of Baouendi and Rothschild in [2; 3] and that of Tumanov in [14] on wedge extendability in *CR* manifolds. We mention that our Lemma 4.2 is similar to Lemma 3.1 of [7].

The paper is organized as follows. In Section 1 we will discuss the locally integrable structures we work in and state our main results. Section 2 contains some corollaries to these results. In Section 3 we recall microlocal hypoanalyticity and prove a lemma concerning the wavefront characterization of the FBI transform. In Section 4 we embed our hypoanalytic structure into a *CR* structure and show that this embedding preserves microlocal

Received March 8, 1991. Revision received August 6, 1992.
Michigan Math. J. 40 (1993).