Möbius Invariant Spaces on the Unit Ball

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In this paper we study spaces of holomorphic functions on the unit ball B in $\mathbf{C}^n$ that are invariant under composition with automorphisms. These spaces are called Möbius invariant, and were introduced by Arazy, Fisher, and Peetre [AFP1]. Previously, Arazy and Fisher [AF1] had proved that on the unit disc D in $\mathbf{C}$ there exists a unique Möbius invariant Hilbert space of holomorphic functions. This turns out to be the classical Dirichlet space $\mathfrak{D}$. Also, Arazy, Fisher, and Peetre constructed a space on the unit disc that is minimal in the class of Möbius invariant Banach spaces. Moreover, in [AFP1] it was proved that, on the unit disc, the minimal space $\mathfrak{M}$ can be identified with the 1-Besov space B_1.

For $n>1$, in [Z1] Zhu proved that there exists a unique Möbius invariant Hilbert space on the unit ball. However, he was not able to find a characterization of this space that extended the Dirichlet space to higher dimensions. This same result was obtained by Peetre, but never published. More recently, Arazy and Fisher [AF2] proved that on any bounded symmetric domain there exists a unique Möbius invariant Hilbert space of holomorphic functions. Again this description is in terms of the power expansion of the holomorphic functions, and a more explicit characterization seems to be desirable. Again in the case $n>1$, Arazy, Fisher, Janson, and Peetre [AFJP] and independently Zhu [Z2] have proved that, for $p>2n$, where n is the dimension of the unit ball, the diagonal Besov spaces $B_p=B_p^{n/p}$ are Möbius invariant.

In this paper we study the Möbius invariant spaces on the unit ball B. We construct a space $\mathfrak{M}$ analogous to the space on the unit disc that we prove to be minimal in the class of Möbius invariant spaces. Moreover, we prove that the space $\mathfrak{M}$ can be identified with the 1-Besov space B_1. As a consequence we obtain that, for $1\le p<\infty$, the Besov spaces B_p are Möbius invariant. Moreover, we prove that the 2-Besov space B_2 is the unique Hilbert space of holomorphic functions that is Möbius invariant, and we give an explicit description of the invariant inner product. Finally, among other properties of the invariant inner product, we prove that the dual of $\mathfrak{M}$ is the Bloch space $\mathfrak{B}$, with equality of norms.

The paper is organized as follows. In Section 1 we give the basic definitions and introduce the Möbius invariant spaces. In Section 2 we construct

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