Common Fixed Points of Commuting Holomorphic Mappings in the Product of n Hilbert Balls

ITAI SHAFRIR

1. Introduction

Let B denote the open unit ball of a complex Hilbert space H. The hyperbolic metric of B is given by the formula,

$$\rho(x,y)=\operatorname{th}^{-1}(1-\sigma(x,y))^{1/2},$$

where $\sigma(x,y)=(1-|x|^2)(1-|y|^2)/|1-(x,y)|^2$ for all $x,y\in B$. More details on the metric space (B,ρ) can be found in the books of Franzoni and Vesentini [FR] and Goebel and Reich [GR].

For $n\geq 1$ consider the hyperball B^n, equipped with its hyperbolic metric,

$$\rho_n(x,y)=\max\{\rho(x_i,y_i);\,1\leq i\leq n\},$$

for all $x=(x_1,\dots,x_n)$ and $y=(y_1,\dots,y_n)$ in B^n. Holomorphic self-mappings of B^n, and more generally ρ_n-nonexpansive mappings, were studied by Kuczumow and Stachura [K1; K2; KS1; KS2], Vigué [V], and Abd-Alla [A1; A2]. In this paper we shall establish the existence of a common fixed point for a family of commuting continuous self-mappings of $\overline{B^n}$ that are holomorphic on B^n. The result provides a positive answer to an open problem of Kuczumow and Stachura [KS2]. Finite-dimensional cases of this result can be found in [S], [E], [HS], and [KS2]. For the result in B ($n=1$), see [K1] or [Si].

2. Preliminaries

In order to understand the geometry of the metric space (B^n,ρ_n), it is useful to study first the space (B,ρ). For each pair of points x,y in B there exists a unique metric segment passing through them. The midpoint of that segment will be denoted by $\frac{1}{2}x\oplus\frac{1}{2}y$; see [GR]. The proof of the next lemma can be found in [Sh].

LEMMA 2.1. *For x,y,z in B,*

$$\rho(\tfrac{1}{2}x\oplus\tfrac{1}{2}y,z)^2\leq\tfrac{1}{2}\rho(x,z)^2+\tfrac{1}{2}\rho(y,z)^2-\tfrac{1}{4}\rho(x,y)^2.$$

Received September 24, 1990. Revision received April 29, 1991.
Michigan Math. J. 39 (1992).