Dual Operator Algebras and a Hereditary Property of Minimal Isometric Dilations

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1. Introduction

Let $\mathcal{H}$ be a separable, infinite dimensional, complex Hilbert space, and let $\mathcal{L}(\mathcal{H})$ be the algebra of all bounded linear operators on $\mathcal{H}$. A *dual algebra* is a subalgebra of $\mathcal{L}(\mathcal{H})$ that contains the identity operator $I_{\mathcal{H}}$ and is closed in the ultraweak operator topology on $\mathcal{L}(\mathcal{H})$. Note that the weak* topology on $\mathcal{L}(\mathcal{H})$ coincides with the ultraweak operator topology on $\mathcal{L}(\mathcal{H})$. The theory of dual algebras is closely related to the study of the classes $\mathbf{A}_{m,n}$ (to be defined below), where m and n are any cardinal numbers such that $1 \leq m, n \leq \aleph_0$. The structures of the classes $\mathbf{A}_{m,n}$ have been applied to the topics of invariant subspaces, dilation theory, and reflexivity (cf. [6]). In particular, the study of these classes has been focused in the last five years on sufficient conditions that a contraction $T \in \mathcal{L}(\mathcal{H})$ belongs to some $\mathbf{A}_{m,n}$. An abstract geometric criterion for membership in $\mathbf{A}_{\aleph_0,\aleph_0}$ was first given in [1]. In a sequel to this study, Brown–Chevreau–Exner–Pearcy (cf. [8], [11], [12], [13]) obtained some relationships between dual algebras and Fredholm theory, and established topological criteria for membership in $\mathbf{A}_{\aleph_0,\aleph_0}$ or $\mathbf{A}_{1,\aleph_0}$. Recently many authors have studied sufficient conditions for membership in the class $\mathbf{A}_{1,\aleph_0}$, $\mathbf{A}_{\aleph_0,\aleph_0}$, or $\mathbf{A}$ (cf. [10], [14], [15], [18]). In particular, in [11] Chevreau–Exner–Pearcy obtained some surprising and unexpected characterizations of the class $\mathbf{A}_{1,\aleph_0}$. As a sequel to these studies, in this note we define a certain hereditary property concerning the minimal isometric dilation of a contraction operator T in $\mathbf{A}$, namely property $(\tilde{\mathbf{H}})$, and show that $T \in \mathbf{A}(\mathcal{H})$ has property $(\tilde{\mathbf{H}})$ if and only if $T \in \mathbf{A}_{1,\aleph_0}$.

2. Notation and Preliminaries

The notation and terminology employed herein agree with that in [2], [6], and [19]. The class $\mathcal{C}_1(\mathcal{H})$ is the Banach space of trace-class operators on $\mathcal{H}$ equipped with the trace norm. The dual algebra $\mathcal{A}$ can be identified with the

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