The Groups of Real Genus 4

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1. Introduction

A finite group G can be represented as a group of automorphisms of a compact bordered Klein surface [14]. In other words, there is a bordered surface on which the group G acts. The *real genus* $\rho(G)$ [14] is the minimum algebraic genus of any bordered Klein surface on which G acts. This parameter is called the "real" genus because of the important correspondence between compact Klein surfaces and real algebraic curves [1]; the bordered surfaces correspond to curves with real points.

The real genus parameter was introduced in [14], and numerous basic results about the parameter were obtained there. In particular, the groups with real genus $\rho \leq 3$ were classified. There are infinite families of groups with $\rho \leq 1$. The groups of real genus 0 are the cyclic and dihedral groups [14, Thm. 3]. The group G has real genus 1 if and only if G is $Z_2 \times D_n$ with n even or $Z_2 \times Z_n$ with n even, $n \geq 4$ [14, Thm. 4]. Interestingly, there are no groups of real genus 2 [14, Thm. 5], and exactly two groups, S_4 and A_4, have real genus 3 [14, Thm. 6]; also see [4], [2], and [3].

Here we classify the groups with real genus 4. Let G_{18} denote the nonabelian group of order 18 that is not D_9 and not $Z_3 \times D_3$. Our main result is the following.

THEOREM 1. *The finite group G has real genus 4 if and only if G is $D_3 \times D_3$, $Z_3 \times D_3$, G_{18}, or $Z_3 \times Z_3$.*

We also develop some general ideas about large groups of automorphisms of bordered surfaces. One consequence is a useful lower bound for the real genus of a group; this lower bound applies to groups with order not divisible by 4 and to groups that cannot be generated by involutions. In addition, we calculate the real genus of two infinite families of supersolvable groups.

2. Preliminaries

We shall assume that all surfaces are compact. Let X be a bordered surface; X is characterized topologically by orientability, the number k of components

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