Weakly and Strongly Outer Functions on the Bidisc

RAYMOND CHENG

0. Introduction

In previous work [1], certain classes of analytic functions on the bidisc were shown to arise naturally in connection with a problem in prediction theory. These were dubbed "weakly outer" and "strongly outer" functions, for it was demonstrated that a regular stationary field has the so-called weak (strong) commutation property if and only if its spectral density is the squared modulus of a weakly (strongly) outer function of the Hardy space H^2. Further applications to prediction were given.

In the present article, the related function theory is explored. We shall see that weakly and strongly outer functions exhibit some properties of the usual outer functions. Indeed, it turns out that, within this context, many of the classical one-variable results have multivariate analogues. Among these are Beurling's theorem, the Riesz factorization, and Szegö's infimum.

1. Preliminaries

Let $\mathbf{D}$ be the unit disc in the complex plane $\mathbf{C}$, and let $\mathbf{T}$ be the unit circle. For $d=1$ or 2, σ_d denotes normalized Lebesgue measure on $\mathbf{T}^d$. We are concerned with the Nevanlinna class $N_*(\mathbf{D}^d)$ as well as the Hardy classes $H^p(\mathbf{D}^d)$ of analytic functions on $\mathbf{D}^d$ (see [3] and [7]). Such a function is associated with its radial limit function on $\mathbf{T}^d$. For convenience, the same letter will be used for both, and corresponding spaces will be identified. The symbol ^ indicates a Fourier coefficient, and C_z represents the Cauchy kernel. Thus, for $d=2$ and $f\in L^1(\mathbf{T}^2)$,

$$\hat{f}(m,n)=\int f(e^{is},e^{it})e^{-ims-int}\,d\sigma_2(e^{is},e^{it});$$

$$C_z=\frac{1}{1-z_1e^{-is}}\cdot\frac{1}{1-z_2e^{-it}}.$$

Received June 7, 1990.
The author was an NSF Graduate Fellow at the University of Virginia when part of this research was performed. This project was also supported in part by NSF Grant DMS 87-01212 at the University of Virginia.
Michigan Math. J. 39 (1992).