On the Structure of Some Lie Algebras of Kuznetsov

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1. Introduction

In [3] Frank showed the existence of two simple graded Lie algebras $L = \sum_{i=-1}^{r} \oplus L_i$ of characteristic 3 for which L_0 is solvable. In [4] Kuznetsov has given a classification of simple graded Lie algebras $L = \sum_{i=-1}^{r} \oplus L_i$ with L_0 having a noncentral radical. In this investigation he encountered algebras of characteristic 3 of what he called series T and $\mathfrak{R}$, which are described in terms of Cartan prolongations of certain solvable Lie algebras.

It is our purpose here to give (in Section 2) a simple description of Kuznetsov's algebras that will reveal many of their structural features, and then in Section 3 to prove the isomorphism of the algebras of series T with those described in [2] and to point out structural differences between those of series $\mathfrak{R}$ and contact algebras of the same dimension.

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2. Description of the Algebras

We shall regard the notation and terminology of [1] as standard and use it without necessarily redefining it here.

Let F be a field of characteristic 3. Let $R = W(2) \oplus \mathfrak{A}(2)$, where $\mathfrak{A}(2)$ is the completed free divided power algebra in x_1, x_2 over F, and $W(2)$ is the Lie algebra of special derivations of $\mathfrak{A}(2)$. R is an algebra under a product $[\ ,\ ]$ such that for $D \in W(2)$ and $f, g \in \mathfrak{A}(2)$, $[D, f] = -[f, D] = \operatorname{div}(fD)$ (where, as usual, $\operatorname{div}(u_1 D_1 + u_2 D_2) = D_1 u_1 + D_2 u_2$); $[f, g] = f\mathfrak{D}_g - g\mathfrak{D}_f$ where $\mathfrak{D}_f = (D_2 f) D_1 - (D_1 f) D_2$; and on $W(2)$ $[\ ,\ ]$ is the usual Lie product.

Let $\mathfrak{A}^{\#}$ be the subset of $\mathfrak{A}(2)$ consisting of all formal sums

$$\sum_{0 \le i \le 1} \sum_{0 \le j} a_{ij} x_1^{(i)} x_2^{(j)} \quad \text{with } a_{ij} \in F,$$

and let $W^{\#}$ be the subalgebra of $W(2)$ consisting of all formal sums

$$\sum_{0 \le i \le 2} \sum_{0 \le j} b_{ij} x_1^{(i)} x_2^{(j)} D_1 + \sum_{0 \le k} c_k x_2^{(k)} D_2 \quad \text{with } b_{ij}, c_k \in F.$$

$T^{\#} = W^{\#} \oplus \mathfrak{A}^{\#}$ is a subalgebra of R under the product $[\ ,\ ]$ defined above.

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