

Bergman Projections and Duality in Weighted Mixed-Norm Spaces of Analytic Functions

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Introduction

In [1], Békollé found a necessary and sufficient condition for weight functions such that the Bergman operator is bounded on the corresponding weighted L^p spaces in the unit ball of \mathbb{C}^N . In [6], Luecking characterized the dual of weighted Bergman spaces with weights satisfying Békollé's conditions. In [5] Jevtić proved that there are bounded projections from general mixed norm spaces onto the weighted mixed norm spaces of analytic functions with the normal-function weights. Using these projections, he characterized the dual of weighted mixed norm spaces of analytic functions with normal-function weights.

In this paper, we show that the Bergman operator is bounded on weighted L^p spaces on the boundary of the unit ball of \mathbb{C}^N with normal-function weights. Then we determine the weighted mixed norm spaces as the interpolation spaces between weighted L^p spaces on the unit ball of \mathbb{C}^N and the weighted L^p spaces on the boundary of the unit ball \mathbb{C}^N with different weights. These facts enable us to prove that the Bergman operator is bounded on weighted mixed norm spaces with radial weights satisfying Békollé's conditions. The characterization of the dual of weighted mixed norm spaces of analytic functions then follows from the boundedness of the Bergman operator.

As applications, we extend Luecking's result to the mixed norm spaces with radial weights, and also show that Jevtić's results are corollaries of our results.

In Section 1, we present the notations and statements of the main results. The proofs of main results are given in Section 2 and Section 3. In the last two sections, applications of the main results are given to extend Luecking's result and to obtain Jevtić's results.

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